

Mathematics

HP COMPUTER CURRICULUM

Mathematical
Systems

TEACHERS ADVISOR

HEWLETT  PACKARD

Hewlett-Packard
Computer Curriculum Series

mathematics
TEACHER'S ADVISOR

**mathematical
systems**

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11000 Wolfe Road
Cupertino, California 95014

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Cupertino, California
Printed in the U.S.A.

The Hewlett-Packard Computer Curriculum Series represents the published results of a Curriculum Development project sponsored by the Data Systems Division of Hewlett-Packard Company. This project is under the directorship of Jean H. Danver.

This material is designed to be used with any Hewlett-Packard system with the BASIC programming language such as the 9830A Educational BASIC, and the 2000 and 3000 series systems.

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INTRODUCTION

This Mathematics Set of the Hewlett-Packard Computer Curriculum Series consists of a set of a Student Lab Book and a corresponding Teacher's Advisor. It was designed to help meet the need for computer-oriented problems in mathematics providing students an opportunity to use a computer as a modeling device within a particular subject matter area.

The materials are designed for flexible use as desired by the individual instructor. The material and exercises in this unit are intended as an "enrichment" experience in the field of mathematical systems. The eleven algebraic properties which hold for the real number system are defined and then applied to three types of systems: sets, matrices, and modular systems. Basic operations for each system are presented; for example, the union and intersection of sets are discussed. The exercises deal with these basic operations and with verifying the applicability of the system properties. Obviously, the unit only scratches the surface of each topic covered, but most of the material will not be covered in your standard text. Thus, it can be used to supplement and enrich your curriculum in any fashion you choose.

The degree of difficulty of the material is dependent upon the amount of assistance which you choose to provide. With no assistance, the better student should be challenged. However, given a good deal of assistance, any second year algebra student should be able to work out the exercises with no great difficulty. The level of the material is determined by the assumption that students taking second year algebra will be quite capable as a group.

The Student Book provides text material and programming exercises for the students. There is a problem analysis, including a suggested approach and a macro flow chart, for each exercise. The mathematical concepts needed for each exercise are briefly reviewed, but you may want your students to study these in greater detail before attempting the exercises, especially if they have no background in sets and matrices. A list of possible references is included at the end of each section. These references will also provide additional problems for your better students. The Teacher's Advisor contains an example of a program to solve each exercise, micro flow charts, and a brief discussion of the important elements of the exercise. The micro flow charts should be given to the students only after they have made an attempt to solve the problem on their own.

For best results, you should study all the solutions until you are certain you have a complete grasp of the general methods. This should be done before assigning the material to the class. Generally, the exercises within each section are cumulative so that as techniques are developed they are used in subsequent exercises. Therefore, you will probably wish to proceed through the exercises in the order in which they are given.

You will undoubtedly think of different programming methods or techniques as you study the example programs. Encourage the students to do the same. There are no *approved* solutions. All solutions are acceptable if they produce the correct results. At this level, there is no need for emphasis on the efficiency of a student's program. The important question is, does it work?

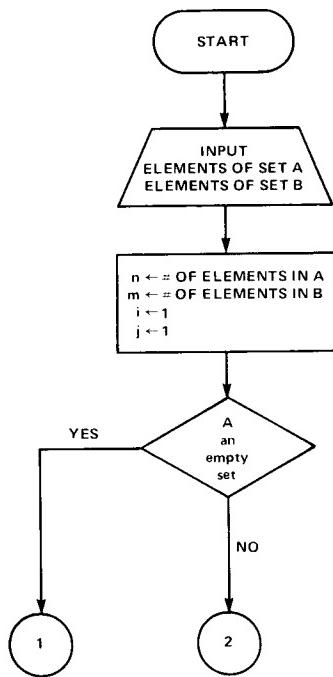
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SETS

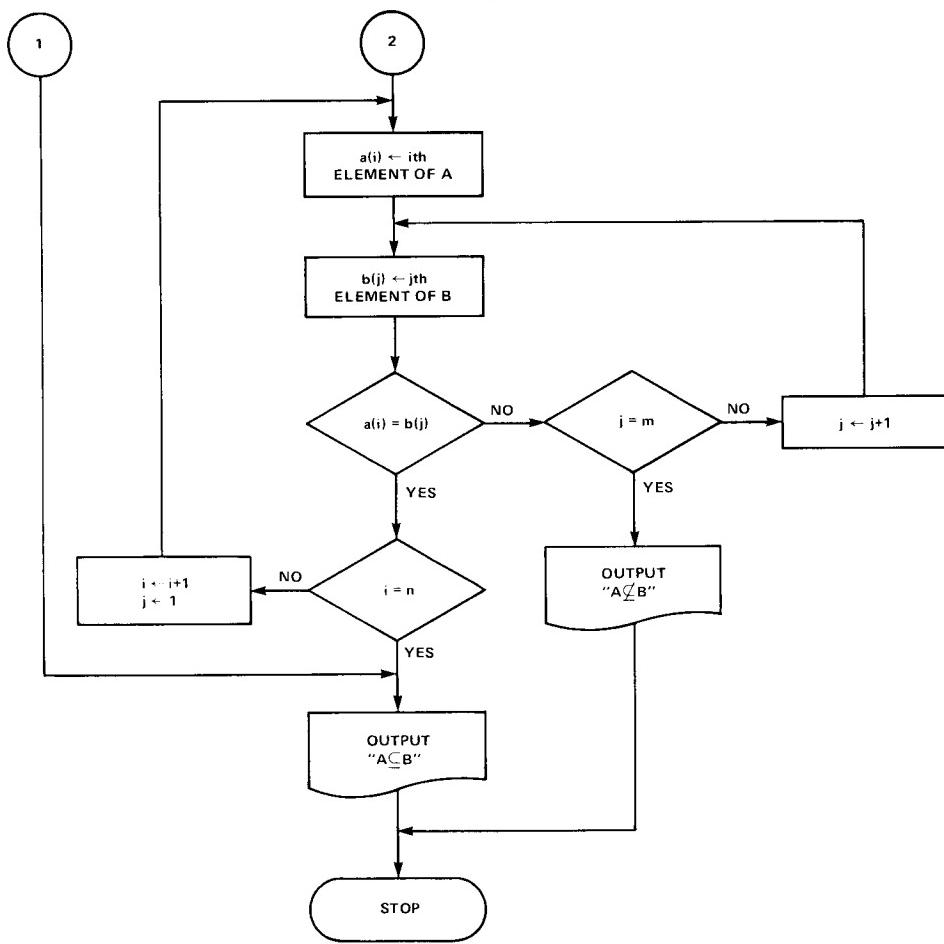
EXERCISE 1 – Determining Subsets

The procedure for this exercise is simple, but the programming is a little tricky. Three conditional loops are involved.

*Micro Flow Chart**Exercise 1*

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*Example Program**Exercise 1*

```

10 REM--THIS PROGRAM WILL DETERMINE IF A FINITE SET A OF REAL
20 REM-- NUMBERS IS A SUBSET OF A FINITE SET B. ON THE DATA LINE
30 REM--ENTER THE NUMBER OF ELEMENTS IN EACH SET FOLLOWED BY THE
40 REM--ELEMENTS OF SET A THEN SET B.
50 READ N,M
60 IF N=0 THEN 130
70 DATA 4,6,7,16,.5,-3,.5,7,-3,13,0,16,3,3,4,.4,13,.4,4,13,5,4,-8
80 DATA 3.1415,10,1,5,-8,3.1415,10,5,4,4,17,-5,2,.01,-7,.1,3,5
90 DATA 0,2,-7,2
100 FOR I=1 TO N
110 READ A[I]
120 NEXT I
130 FOR J=1 TO M
140 READ B[J]
150 NEXT J
160 IF N=0 THEN 300
170 I=1
180 J=1
190 IF A[I]=B[J] THEN 230
200 IF J=M THEN 270
210 J=J+1
220 GOTO 190
230 IF I=N THEN 300
240 I=I+1
250 J=1
260 GOTO 190
270 PRINT "A IS NOT A SUBSET OF B"
280 PRINT
290 GOTO 50
300 PRINT "A IS A SUBSET OF B"
310 PRINT
320 GOTO 50
330 END
RUN

```

A IS A SUBSET OF B

A IS A SUBSET OF B

A IS NOT A SUBSET OF B

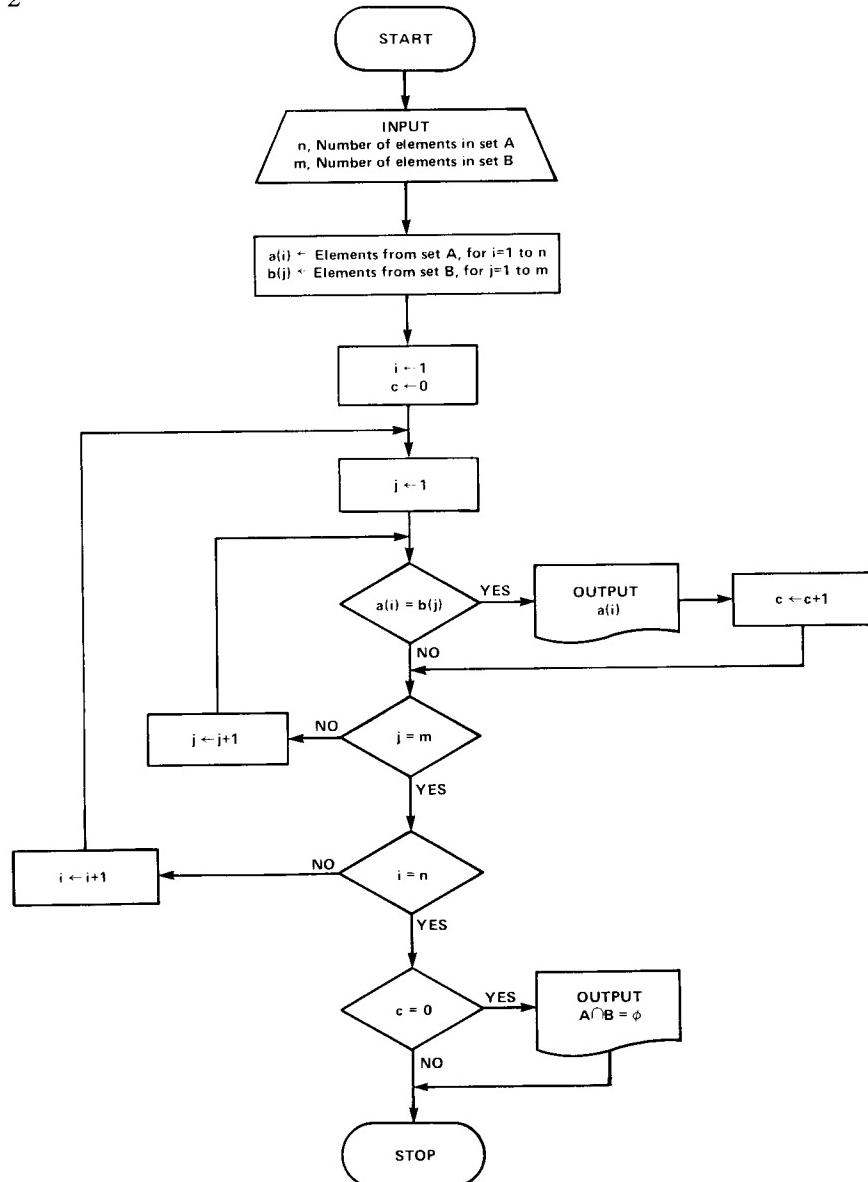
A IS NOT A SUBSET OF B

A IS A SUBSET OF B

OUT OF DATA IN LINE 50

EXERCISE 2 – The Intersection of Sets

This exercise models the definition of intersection. The value of the problem comes not from the significance of its application but rather from the modeling experience.

*Micro Flow Chart**Exercise 2*

Example Program

Exercise 2

```
10 REM--THIS PROGRAM WILL DETERMINE THE INTERSECTION SET OF TWO
20 REM--NON-EMPTY SETS, A AND B, OF REAL NUMBERS.
40 LET C=0
50 DATA 4,3,-2.5,7,.27,4,-.7,3,-6,.5,3,17,3.14,-17,5,0,17,-6
60 DATA 3.14,-17,4,7,-.66,25,6,3,-2,0,2.16,3,-5,0,1,3,0,-5,1
70 READ N
75 PRINT "A INTERSECT B =";
80 FOR I=1 TO N
90 READ A[I]
100 NEXT I
110 READ M
120 FOR J=1 TO M
130 READ B[J]
140 NEXT J
150 FOR I=1 TO N
160 FOR J=1 TO M
170 IF A[I]=B[J] THEN 220
180 NEXT J
190 NEXT I
200 IF C=0 THEN 250
205 PRINT
206 PRINT
210 GOTO 40
220 LET C=C+1
230 PRINT A[I];
240 GOTO 190
250 PRINT "THE NULL SET"
251 PRINT
252 PRINT
255 GOTO 40
260 END
RUN
```

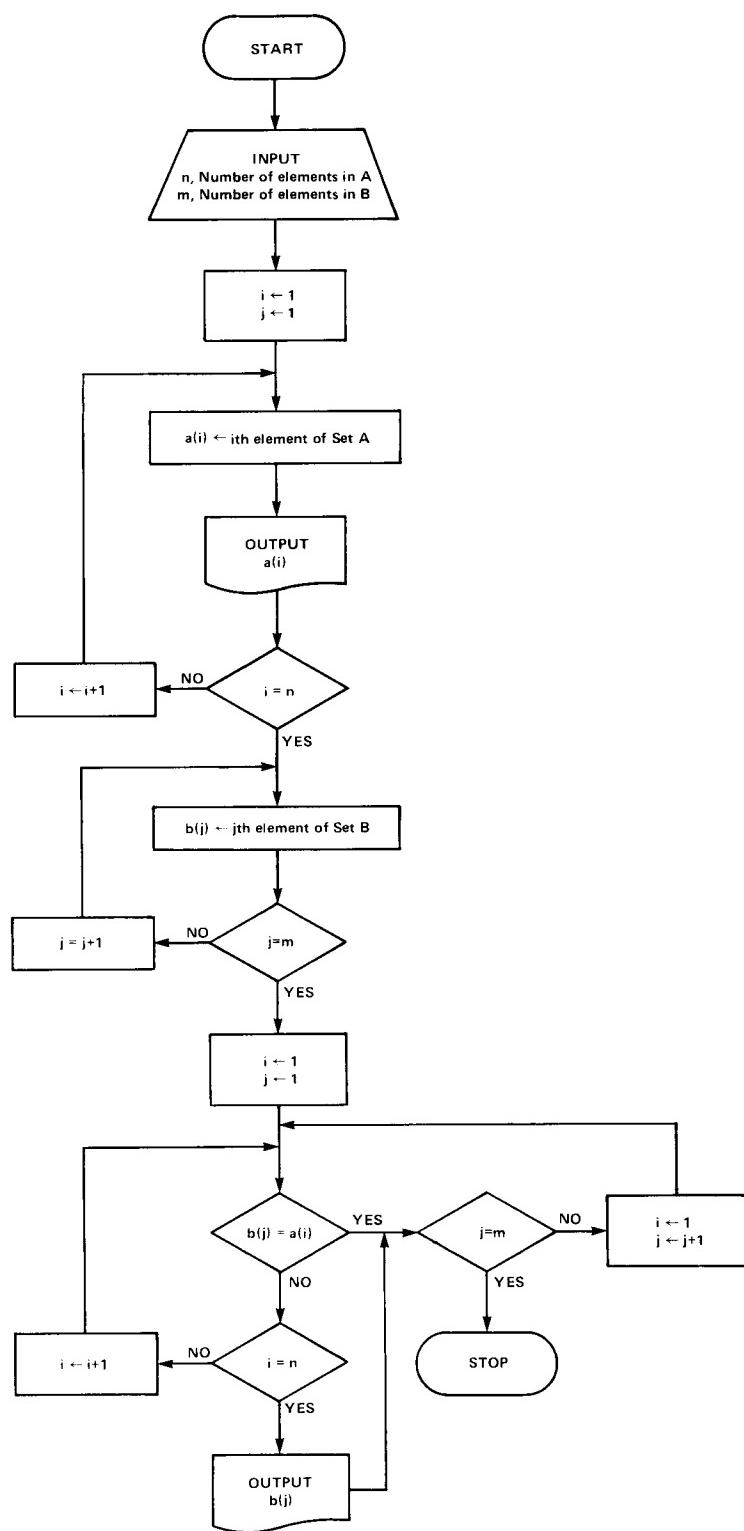
A INTERSECT B = 3

A INTERSECT B = 17 3.14 -17

A INTERSECT B =THE NULL SET

A INTERSECT B =-5 0 1

OUT OF DATA IN LINE 70

EXERCISE 3 – The Union of Sets*Micro Flow Chart*

Example Program

Exercise 3

```
10 REM-- A PROGRAM TO FORM THE UNION OF TWO NON-EMPTY SETS OF
20 REM-- NUMBERS. THE DATA LINE SHOULD CONTAIN THE NUMBER OF
40 REM-- ELEMENTS IN THE SECOND SET. THEN THE ELEMENTS OF
50 REM-- OF THE FIRST SET AND THE ELEMENTS OF THE SECOND SET
60 REM-- IN THAT ORDER.
70 READ N,M
75 PRINT "THE UNION OF SET A WITH SET B CONTAINS ELEMENTS:";
80 FOR I=1 TO N
90 READ A[I]
95 PRINT A[I];
100 NEXT I
110 FOR J=1 TO M
120 READ B[J]
130 NEXT J
140 I=1
150 J=1
160 IF B[J]=A[I] THEN 200
170 IF I=N THEN 230
180 I=I+1
190 GOTO 160
200 IF J=M THEN 300
205 I=1
210 J=J+1
220 GOTO 160
230 PRINT B[J];
240 GOTO 200
250 DATA 5,5,6,7,-2,-1.6,75,3,-18,-2,75,11
300 END
RUN
```

```
THE UNION OF SET A WITH SET B CONTAINS ELEMENTS: 6      7      -2
-1.6      75      3      -18      11
```

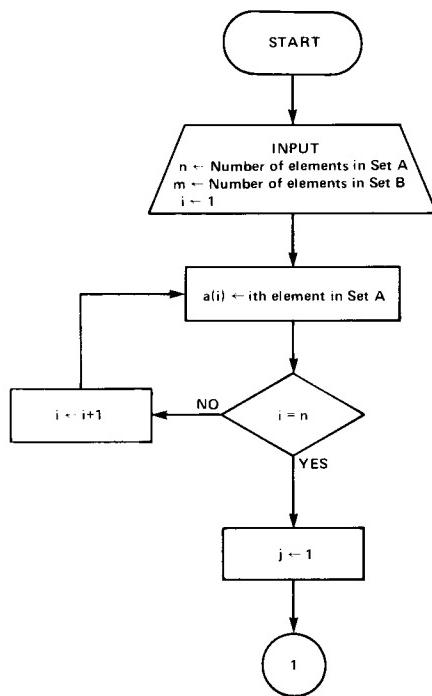
```
250 DATA 5,5,7,-7,0,1,-1,7,-7,0,1,-1
RUN
```

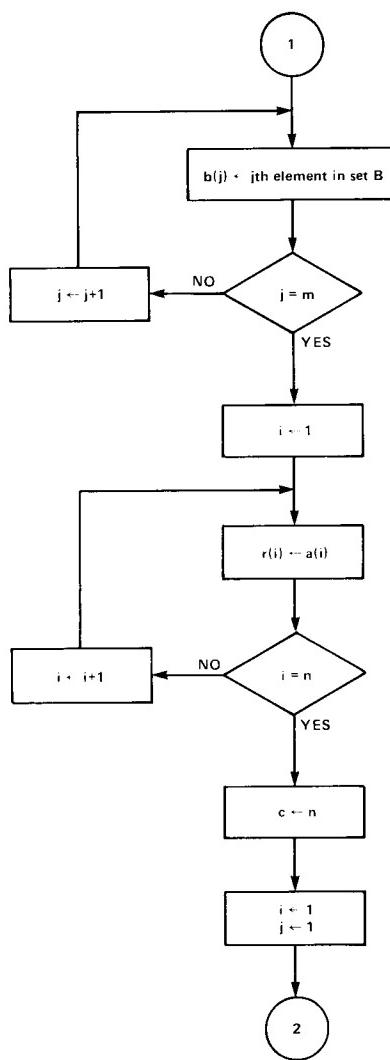
```
THE UNION OF SET A WITH SET B CONTAINS ELEMENTS: 7      -7      0
-1
```

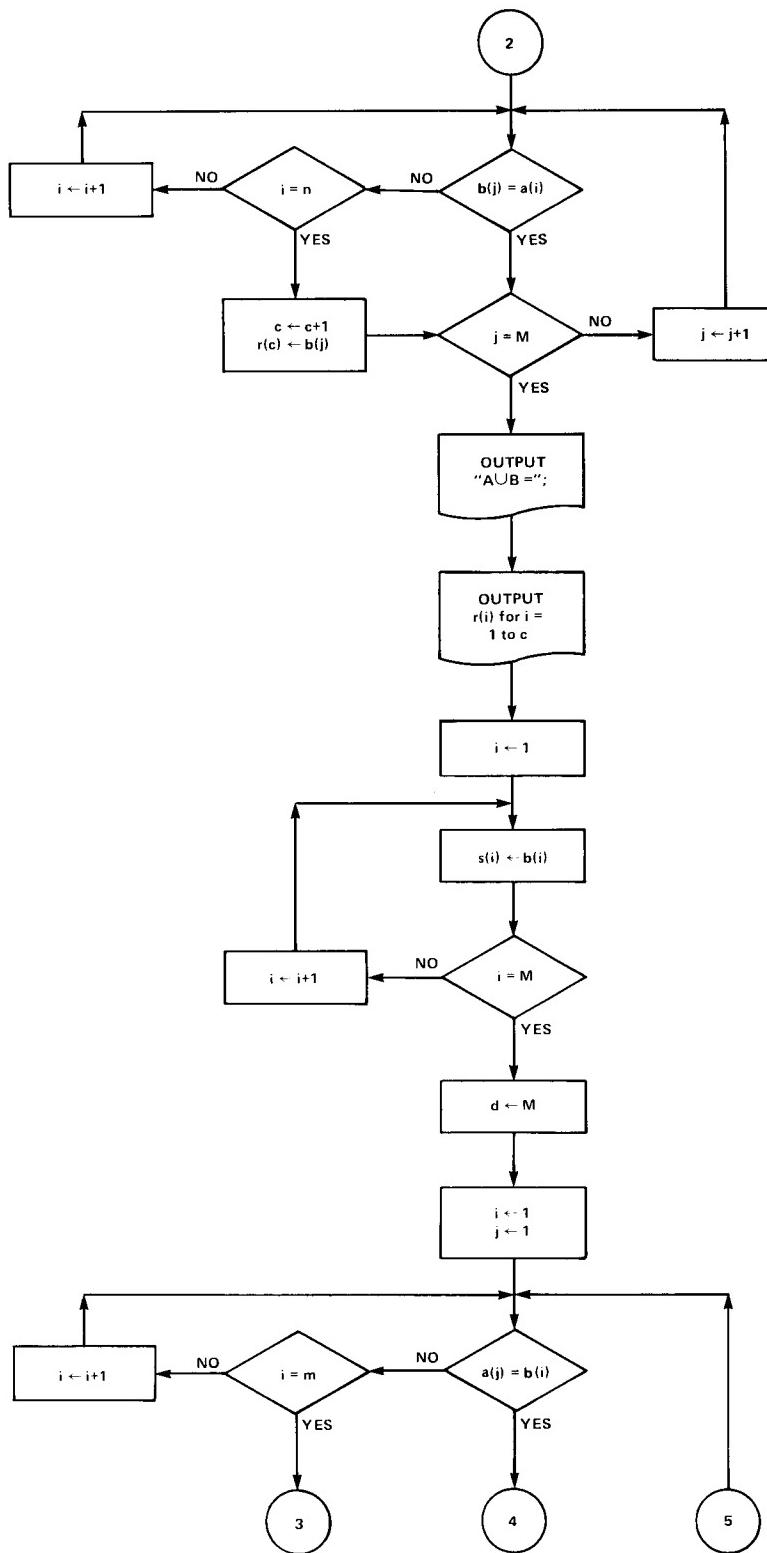
EXERCISE 4 – Modeling Commutativity of the Union Operation For Two Given Sets A and B

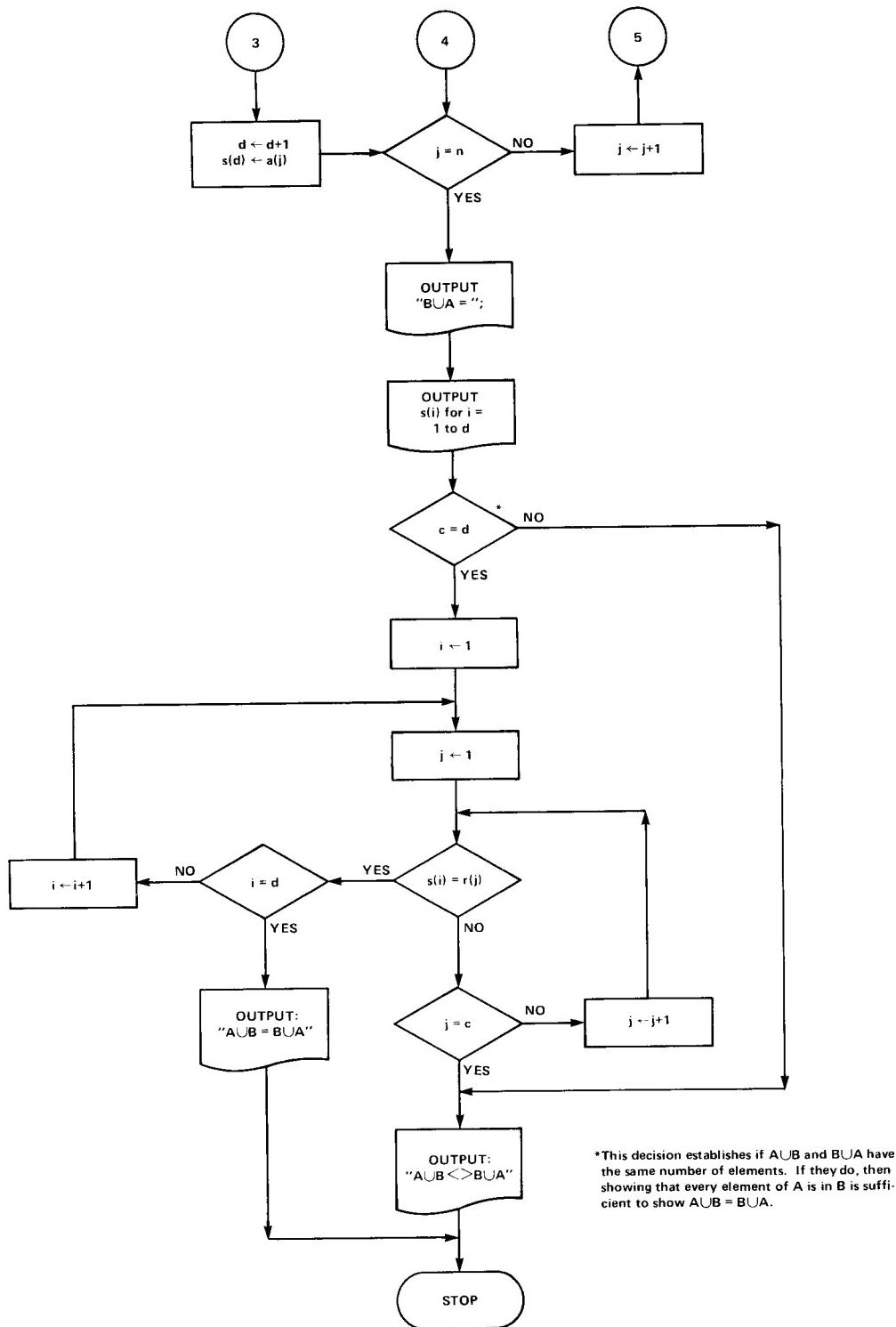
This is a good exercise in flow charting. If the flow charting is done correctly, the BASIC program easily follows.

You might also point out to students that writing formal proofs of generalization is not so bad when you consider the work involved in checking this one instance. Of course, a paper and pencil check would be rather easy if the number of elements in A or B is not excessive.

Micro Flow Chart







*This decision establishes if $A \cup B$ and $B \cup A$ have the same number of elements. If they do, then showing that every element of A is in B is sufficient to show $A \cup B = B \cup A$.

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Example Program

Exercise 4

```
10 REM--THIS PROGRAM DETERMINES IF THE "UNION" OPERATIONS
11 REM--ON TWO GIVEN SETS A AND B IS COMMUTATIVE. ON THE DATA
12 REM--LINE THE OPERATOR MUST PROVIDE THE NUMBER OF ELEMENTS IN THE SE
13 REM--A, THE NUMBER OF ELEMENTS IN THE SET B, THE ELEMENTS OF SET A
14 REM--AND THE ELEMENTS OF SET B IN THAT ORDER.
20 READ N,M
30 FOR I=1 TO N
40 READ A[I]
50 NEXT I
60 FOR J=1 TO M
70 READ B[J]
80 NEXT J
90 FOR I=1 TO N
100 R[I]=A[I]
110 NEXT I
120 C=N
130 FOR J=1 TO M
140 FOR I=1 TO N
150 IF B[J]=A[I] THEN 190
160 NEXT I
170 LET C=C+1
180 LET R[C]=B[J]
190 NEXT J
200 PRINT "A U B =";
205 FOR I=1 TO C
210 PRINT R[I];
220 NEXT I
230 PRINT
231 FOR I=1 TO M
240 S[I]=B[I]
250 NEXT I
260 D=M
270 FOR J=1 TO N
280 FOR I=1 TO M
290 IF A[J]=B[I] THEN 310
300 NEXT I
303 LET D=D+1
305 LET S[D]=A[J]
310 NEXT J
315 PRINT "B U A =";
320 FOR I=1 TO D
330 PRINT S[I];
335 NEXT I
340 PRINT
350 PRINT
370 IF C=D THEN 400
380 PRINT "THEREFORE A U B <=> B U A"
385 PRINT ""
390 GOTO 20
400 FOR I=1 TO D
410 FOR J=1 TO C
420 IF S[I]=R[J] THEN 450
430 NEXT J
440 GOTO 380
450 NEXT I
470 PRINT "THERFORE A U B = B U A"
473 PRINT ""
475 GOTO 20
480 DATA 5,5,6,7,-2,-1,6,75,3,-18,-2,75,11,5,5,7,-7,0,1,-1,7,-7,0,1,-1
490 END
```

RUN

$$\begin{array}{l} A \cup B = 6 \quad 7 \quad -2 \quad -1.6 \quad 75 \quad 3 \quad -18 \quad 11 \\ B \cup A = 3 \quad -18 \quad -2 \quad 75 \quad 11 \quad 6 \quad 7 \quad -1.6 \end{array}$$

THERFORE $A \cup B = B \cup A$

$$\begin{array}{l} A \cup B = 7 \quad -7 \quad 0 \quad 1 \quad -1 \\ B \cup A = 7 \quad -7 \quad 0 \quad 1 \quad -1 \end{array}$$

THERFORE $A \cup B = B \cup A$

OUT OF DATA IN LINE 20

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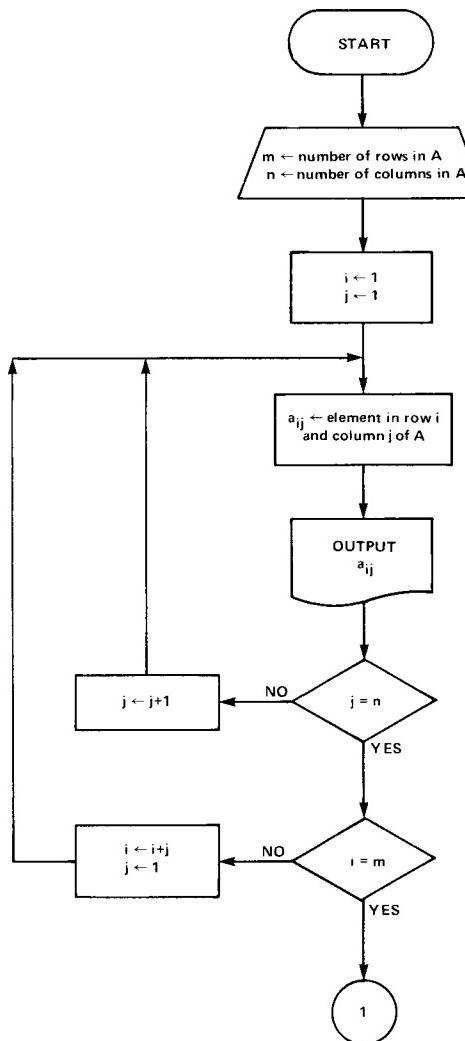
MATRICES

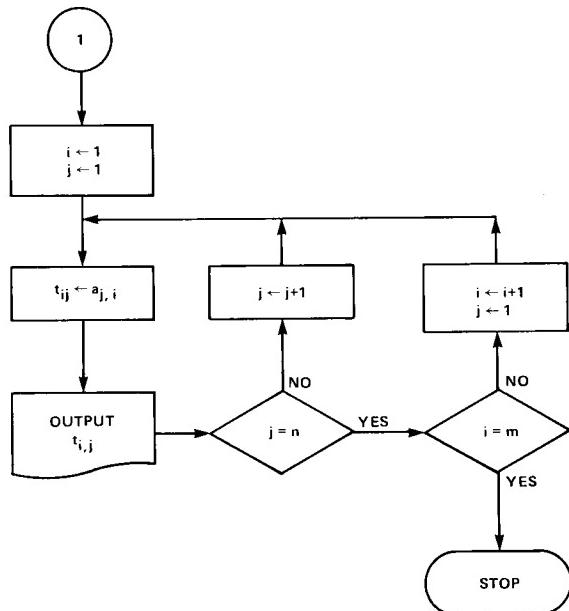
EXERCISE 5 – Modeling Matrix Definitions

This exercise tests the student's understanding of the basic definitions in matrix algebra while providing experience in manipulating elements within a matrix.

Micro Flow Chart

Exercise 5(a)





Example Program

Exercise 5(a)

```

10 REM--A PROGRAM TO PRINT THE TRANSPOSE OF A MATRIX.
20 REM--AND THE ELEMENTS OF THE MATRIX READING FROM LEFT TO
30 REM--RIGHT ROW BY ROW
40 PRINT
50 READ N,M
60 PRINT "MATRIX A IS"
70 PRINT
80 FOR I=1 TO M
90 FOR J=1 TO N
100 READ A[I,J]
110 PRINT TAB(11*N);A[I,J];
120 NEXT J
130 PRINT
140 NEXT I
150 PRINT
160 PRINT
170 PRINT
180 PRINT "THE TRANSPOSE OF A IS"
190 PRINT
200 FOR I=1 TO M
210 FOR J=1 TO N
220 LET T[I,J]=A[J,I]
230 PRINT TAB(11*N);T[I,J];
240 NEXT J
250 PRINT
260 NEXT I
270 DATA 4,4,1,-7,0,8,3,0.5,-3,7,0,1,1.5,23,8,2,5,-0.8
280 END
  
```

RUN

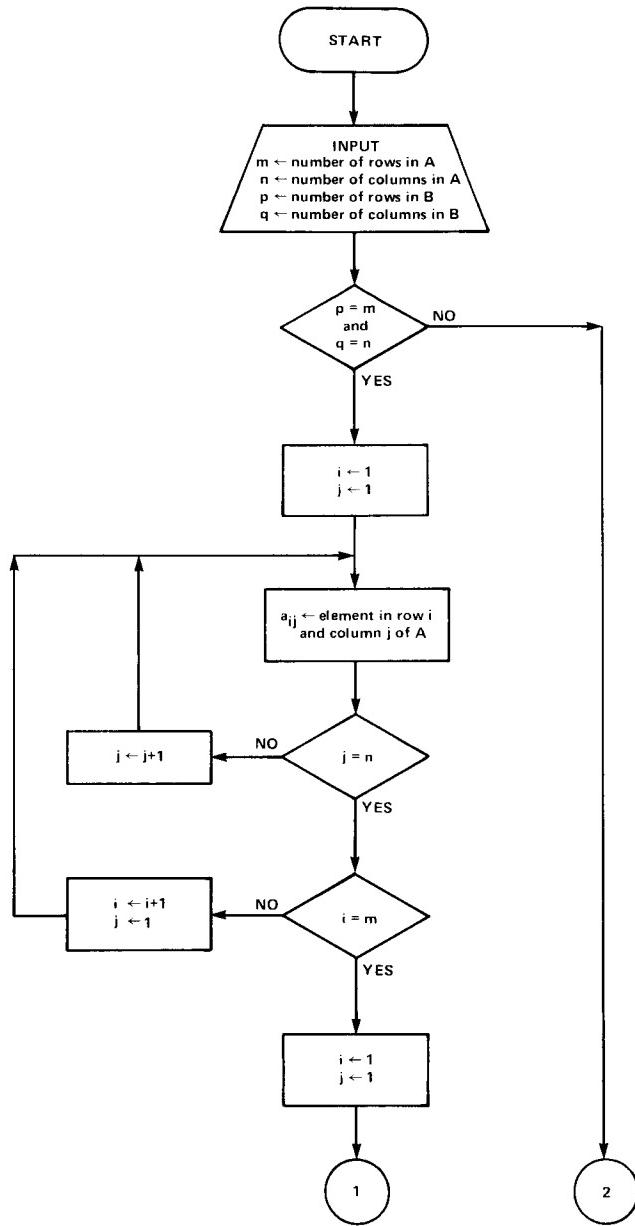
MATRIX A IS

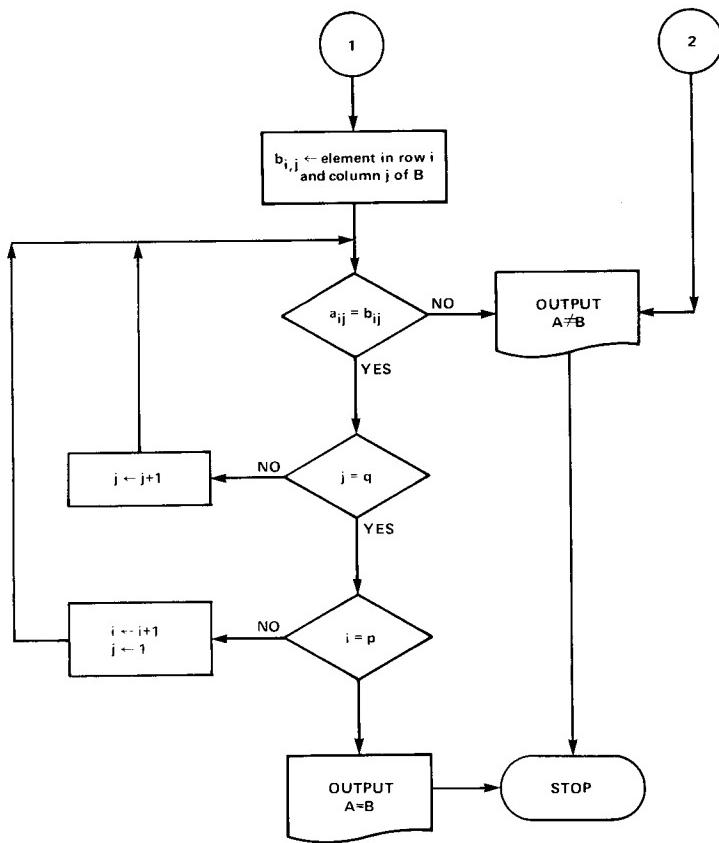
1	-7	0	8
3	0.5	-3	7
0	1	15	23
8	2	5	-0.8

THE TRANSPOSE OF A IS

1	3	0	8
-7	0.5	1	2
0	-3	15	5
8	7	23	-0.8

DONE

*Micro Flow Chart**Exercise 5(b)*



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Example Program

Exercise 5(b)

```
10 REM--A PROGRAM TO DETERMINE IF TWO GIVEN MATRICES ARE EQUAL.
20 REM--ON THE DATA LINE ENTER THE ORDER M,N AND P,Q OF MATRICES
30 REM--A AND B RESPECTIVELY. THEN ENTER THE ELEMENTS OF A AND B
40 REM--ROW BY ROW.
50 READ M,N,P,Q
60 IF M <> P THEN 210
70 IF N <> Q THEN 210
80 FOR I=1 TO M
90 FOR J=1 TO N
100 READ A[I,J]
110 NEXT J
120 NEXT I
130 FOR I=1 TO P
140 FOR J=1 TO Q
150 READ B[I,J]
160 IF A[I,J] <> B[I,J] THEN 210
170 NEXT J
180 NEXT I
190 PRINT "A = B"
200 GOTO 300
210 PRINT "B <> A"
220 GOTO 240
230 DATA 2,3,3,3,3,2,-1,6,8,0,3,2,-1,6,8,0,-5,6,.6
300 END
RUN
B <> A
DONE
```

```
230 DATA 3,3,3,3,3,2,-1,6,8,0,-5,7,.6,3,2,-1,6,8,0,-5,6,.6
RUN
```

B <> A

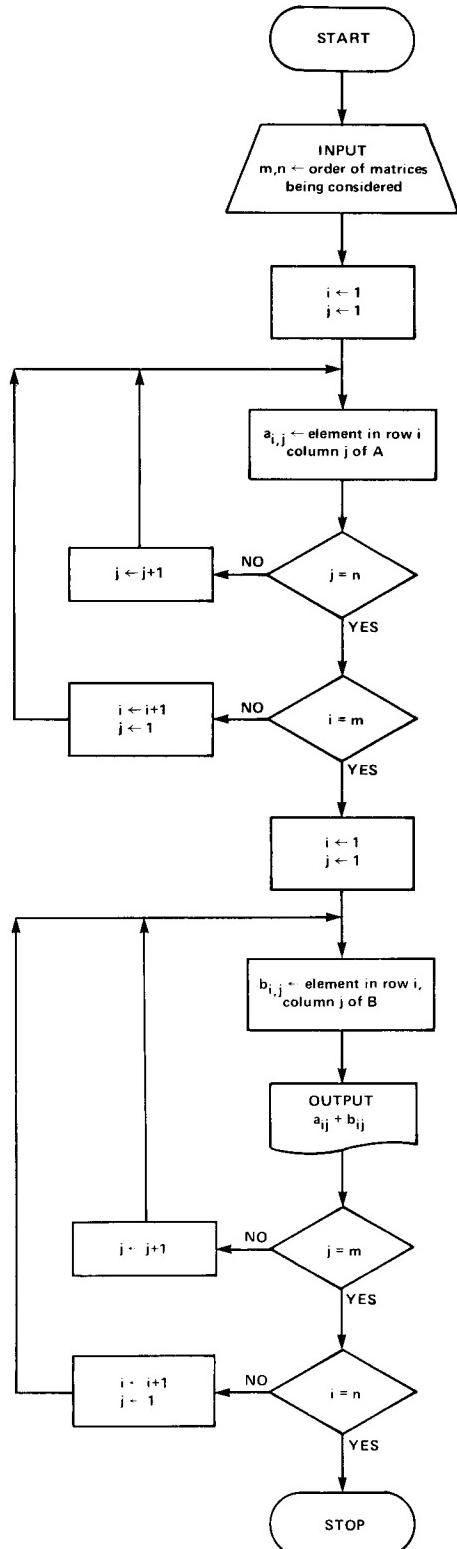
DONE

```
230 DATA 4,4,4,4,7,-2,.5,1,6,0,-7,0,-.7,2,1,3,12,8,19,-20,7,-2
235 DATA .5,1,6,0,-7,0,-.7,2,1,3,12,8,19,-20
RUN
```

A = B

Micro Flow Chart

Exercise 5(c)



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Example Program

Exercise 5(c)

```
10 REM--THIS PROGRAM MODELS THE DEFINITION OF THE SUM OF TWO
20 REM--MATRICES A AND B. ENTER ON THE DATA LINE
30 REM-- THE ORDER OF THE THE MATRICES M,N AND THE ELEMENTS OF
40 REM--OF MATRIX A AND MATRIX B READING FROM LEFT TO RIGHT
50 REM--ROW BY ROW.
60 READ M,N
70 FOR I=1 TO M
80 FOR J=1 TO N
90 READ A[I,J]
100 NEXT J
110 NEXT I
120 FOR I=1 TO M
130 FOR J=1 TO N
140 READ B[I,J]
150 PRINT A[I,J]+B[I,J];
160 NEXT J
170 PRINT
180 NEXT I
190 PRINT
200 PRINT
210 PRINT
220 GOTO 60
230 DATA 4,3,8,-2,5,-20,.5,1,0,-7,.8,6,1,0,16,27,0,-1,18,2,5
240 DATA -.16,3,0,1,7,2,4,3,-7,1,.8,2,7,0,5,17,7,0,.2,3,3,10,5
250 END
RUN
```

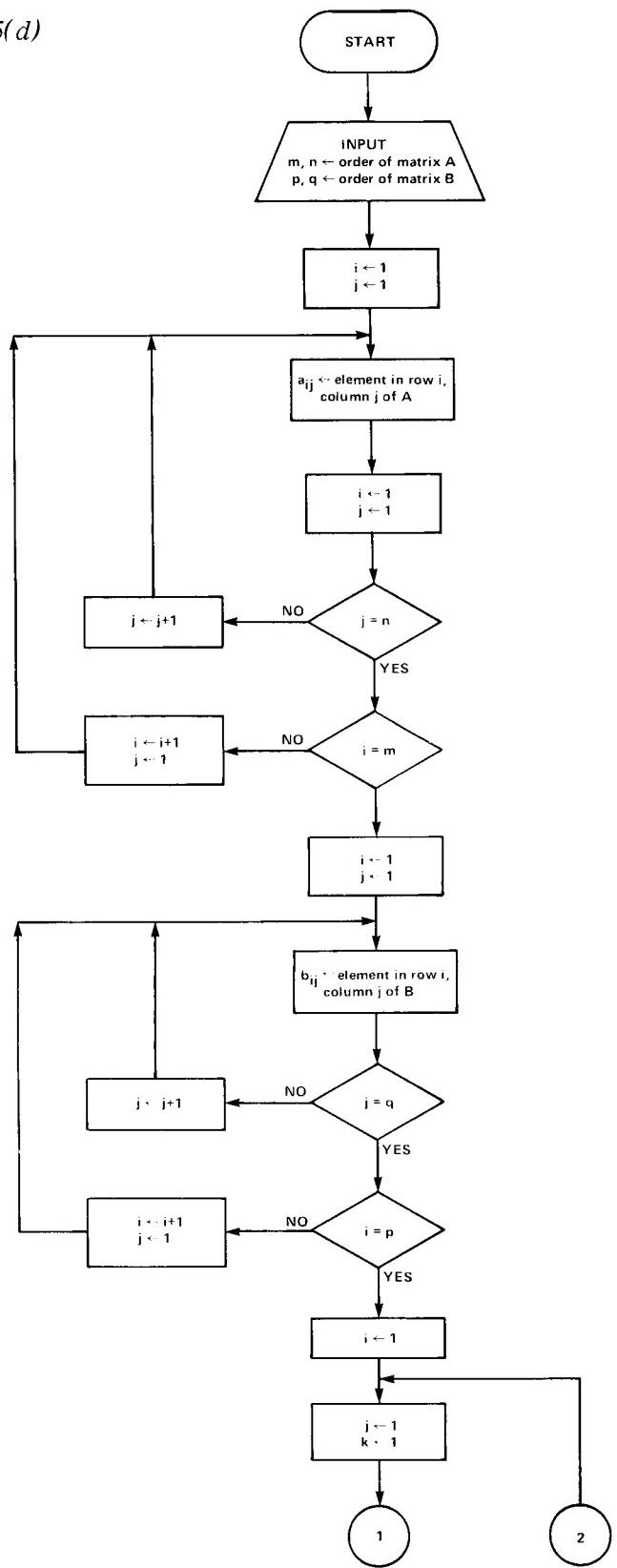
```
24      25      5
-21     18.5      3
5      -7.16      3.8
6        2      7
```

```
20      0       1       1
5      10      10      10
```

OUT OF DATA IN LINE 60

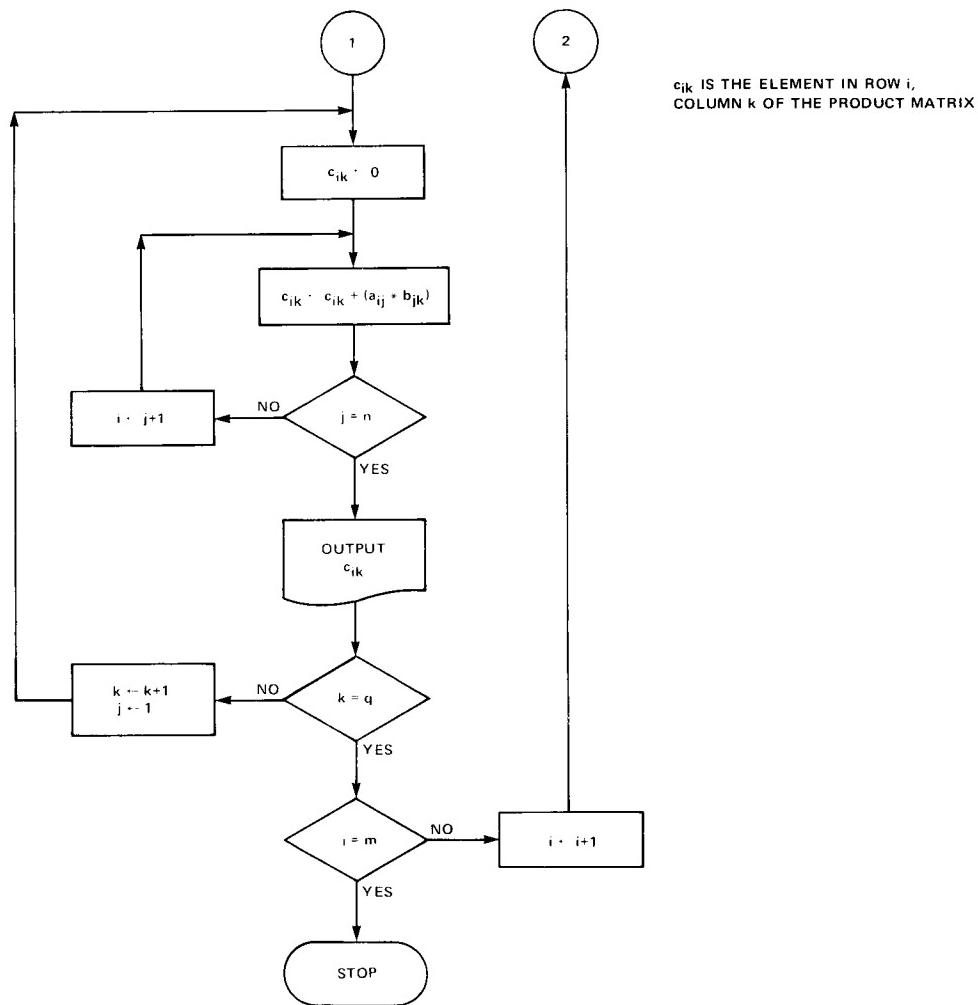
Micro Flow Chart

Exercise 5(d)



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*Example Program**Exercise 5(d)*

```

10 REM--A PROGRAM TO MODEL MATRIX MULTIPLICATION.
20 REM--ON THE DATA LINE ENTER M,N AND P,Q, THE ORDER OF THE
30 REM--MATRICES A AND B RESPECTIVELY. THEN ENTER THE ELEMENTS
40 REM--OF THE MATRICES, ROW BY ROW.
50 READ M,N,P,Q
60 PRINT
70 FOR I=1 TO M
80 FOR J=1 TO N
90 READ A[I,J]
100 NEXT J
110 NEXT I
120 FOR I=1 TO P
130 FOR J=1 TO Q
140 READ B[I,J]
150 NEXT J
160 NEXT I
170 I=1
180 J=K=1
190 C[I,K]=0
200 C[I,K]=C[I,K]+A[I,J]*B[J,K]
210 IF J=N THEN 240
220 J=J+1
230 GOTO 200
240 PRINT C[I,K];
250 IF K=Q THEN 290
260 K=K+1
270 J=1
280 GOTO 190
290 PRINT
300 IF I=M THEN 50
310 I=I+1
320 GOTO 180
330 DATA 3,3,3,3,-3,0,1,2,1,3,-2,0,1,2,1,-1,2,3,1,-2,1,0,2,3
340 DATA 3,2,3,-2,1,2,0,-5,2,-5,0,1,2,2,2,2,2,1,1,1,-1,-1,2
350 END
RUN

```

-8 -2 3
 0 8 -1
 -6 -1 2

8 -15
 -6 -20

1 0
 0 1

OUT OF DATA IN LINE 50

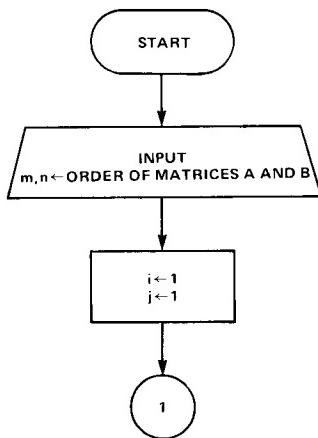
EXERCISE 6 – Properties of a System of Square Matrices

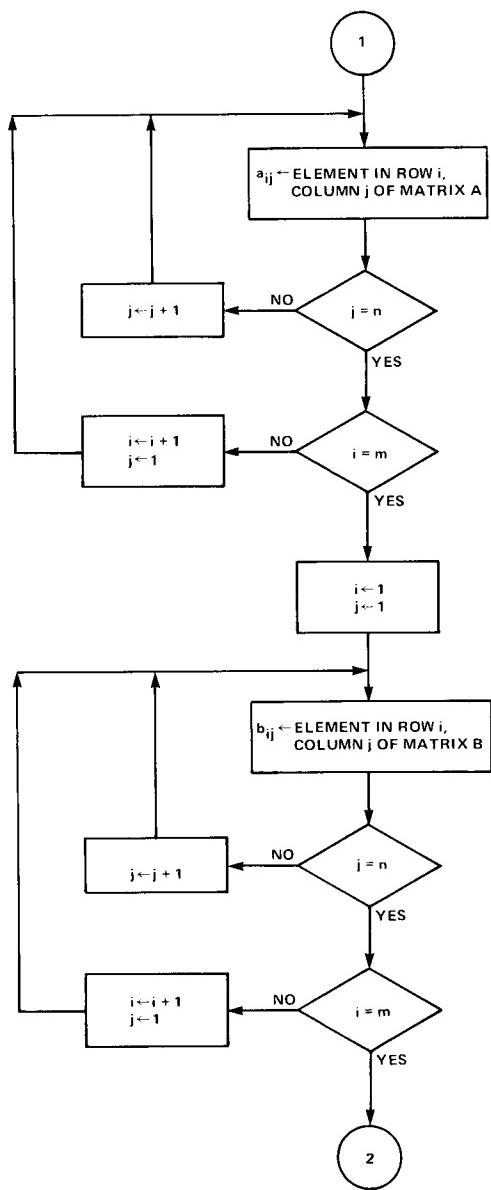
Although the computer system you are using may have matrix manipulation capabilities, it is important that your students understand the processes involved in each operation. This exercise requires that the students be familiar with the basic operations and properties of matrix algebra.

- (a) In this part of the exercise, be sure to emphasize that because commutativity holds for a finite number of cases does not necessarily prove that it holds for all cases.

Micro Flow Chart

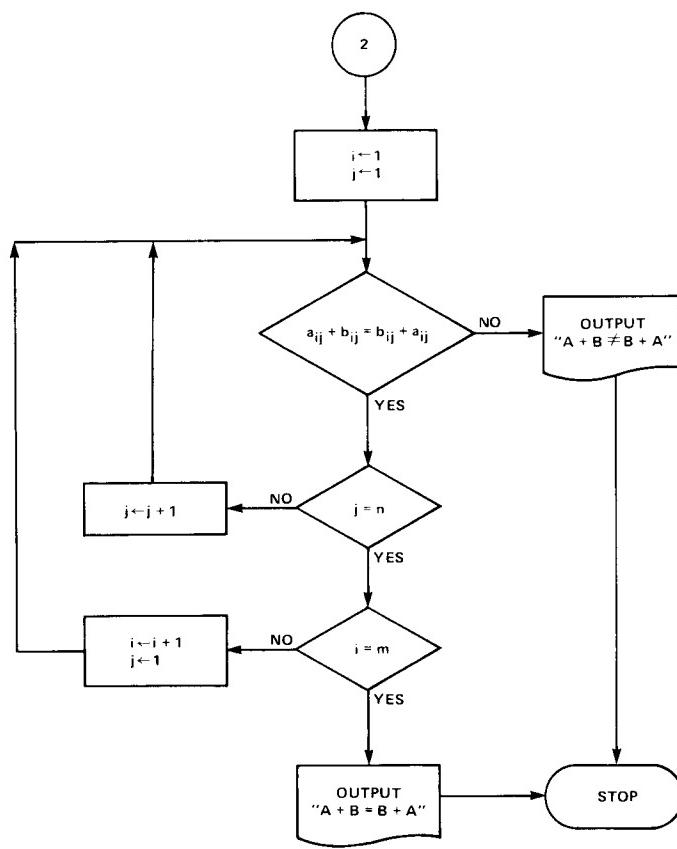
Exercise 6(a)





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*Example Program**Exercise 6(a)*

```

10 REM--THIS PROGRAM ILLUSTRATES THE COMMUTATIVE PROPERTY FOR MATRIX
20 REM--ADDITION. AN INPUT OF THE ORDER OF THE MATRIX IS REQUIRED
30 REM--THE DATA LINE SHOULD CONTAIN THE ELEMENTS OF MATRICES A AND B
40 DATA 8,-2,5,20,.5,1,0,-7,.8,6,1,0
50 DATA 16,27,0,-1,18,2,5,-16,3,0,1,7
60 PRINT "WHAT IS THE ORDER OF THE MATRICES INVOLVED":;
70 INPUT M,N
80 FOR I=1 TO M
90 FOR J=1 TO N
100 READ A[I,J]
110 NEXT J
120 NEXT I
130 FOR I=1 TO M
140 FOR J=1 TO N
150 READ B[I,J]
160 NEXT J
170 NEXT I
180 FOR I=1 TO M
190 FOR J=1 TO N
200 IF A[I,J]+B[I,J]=B[I,J]+A[I,J] THEN 230
210 PRINT "A+B <> B+A FOR MATRICES A AND B."
220 GOTO 260
230 NEXT J
240 NEXT I
250 PRINT "A+B=B+A FOR MATRICES A AND B."
260 END
RUN

```

WHAT IS THE ORDER OF THE MATRICES INVOLVED?4,3
 A+B=B+A FOR MATRICES A AND B.

DONE

```

40 DATA 3,-7,1,.8,2,7,0,5
50 DATA 17,7,0,.2,3,3,10,5
RUN

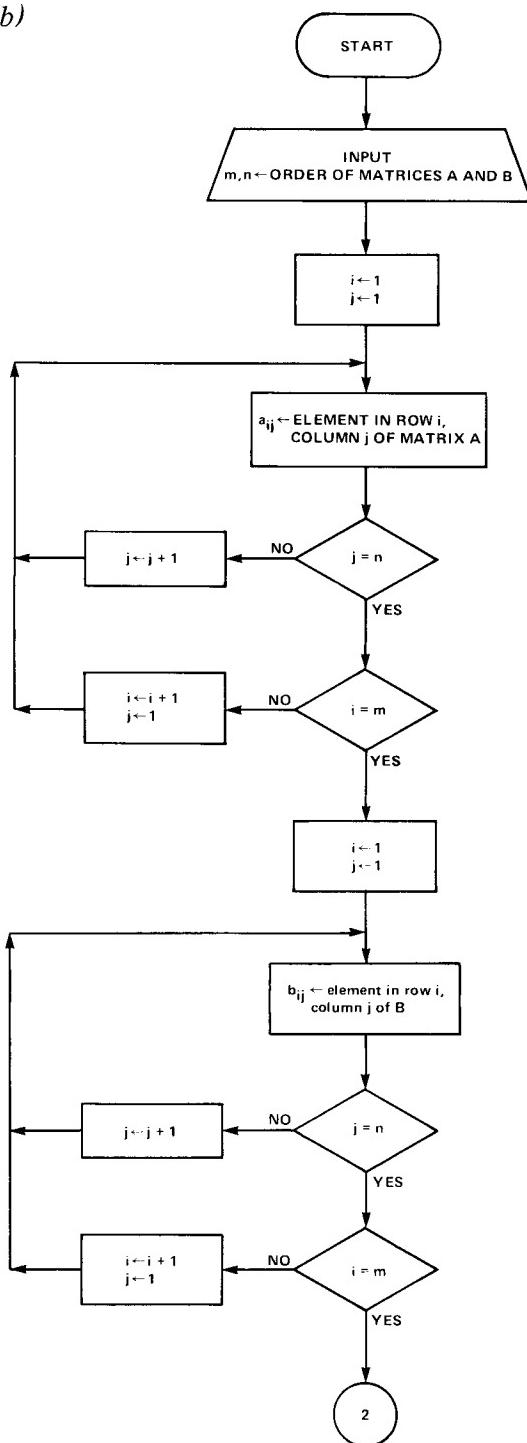
```

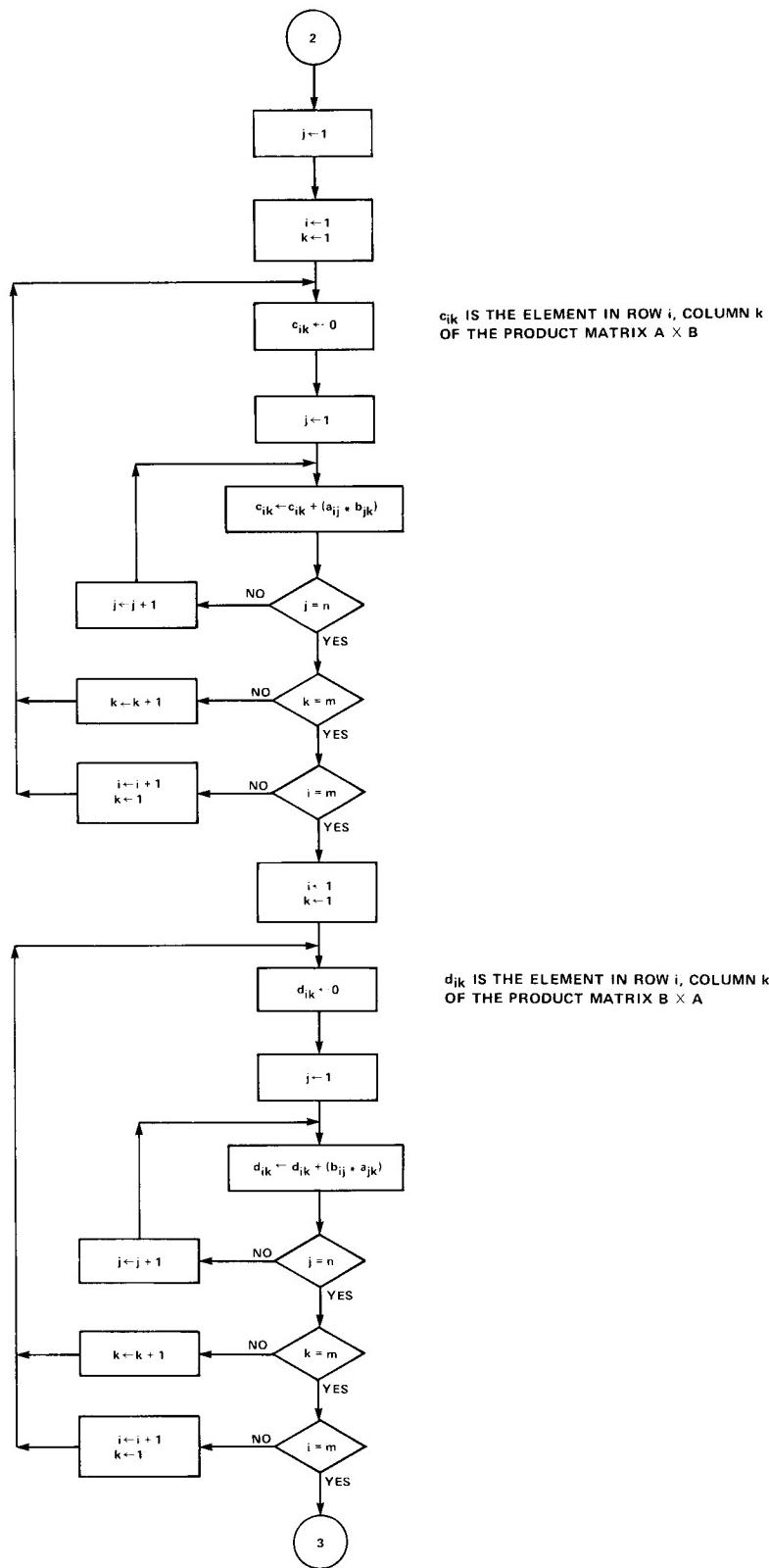
WHAT IS THE ORDER OF THE MATRICES INVOLVED?4,2
 A+B=B+A FOR MATRICES A AND B.

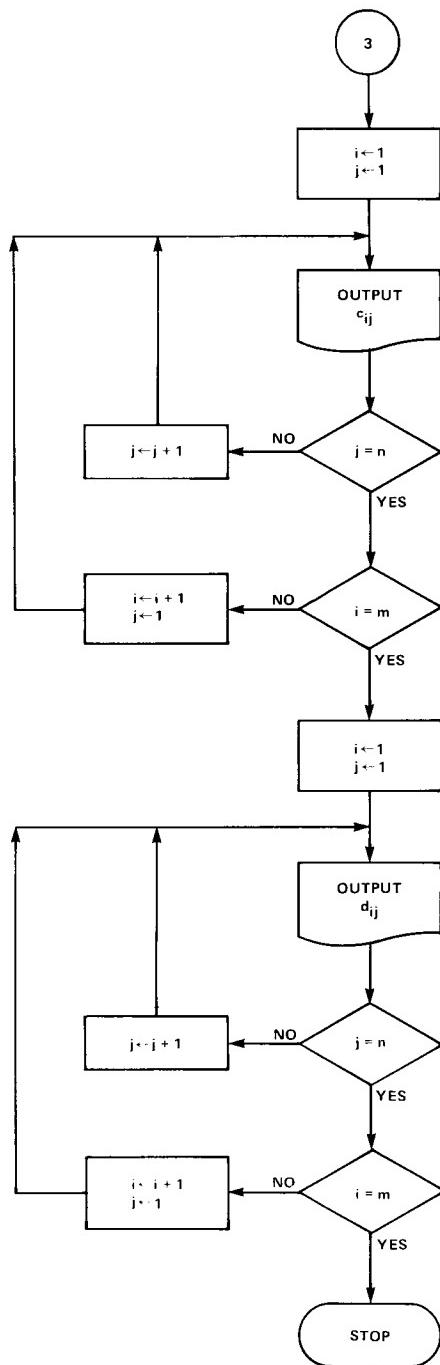
DONE

Micro Flow Chart

Exercise 6(b)







*Example Program**Exercise 6(b)*

```

10 REM--THIS PROGRAM PRINTS OUT THE PRODUCTS A X B AND B X A
11 REM--FOR TWO MATRICES A AND B TO ILLUSTRATE THAT MATRIX
12 REM-- MULTIPLICATION IS NON COMMUTATIVE. LINE 20 REQUIRES THE INPUT
13 REM--OF THE ORDER OF THE MATRICES. THE DATA LINE SHOULD
14 REM--CONTAIN THE ELEMENTS OF A AND B IN THAT ORDER.
20 INPUT M,N
30 DATA 6,-2,1,0,1,-1,3,2,1,-2,3,0,4,5,3,-1,0,1
40 FOR I=1 TO M
50 FOR J=1 TO N
60 READ A[I,J]
70 NEXT J
80 NEXT I
90 FOR I=1 TO M
100 FOR J=1 TO N
110 READ B[I,J]
120 NEXT J
130 NEXT I
140 FOR I=1 TO M
145 FOR K=1 TO M
146 C[I,K]=0
150 FOR J=1 TO N
160 C[I,K]=C[I,K]+A[I,J]*B[J,K]
170 NEXT J
175 NEXT K
180 NEXT I
190 FOR I=1 TO M
195 FOR K=1 TO M
196 D[I,K]=0
200 FOR J=1 TO N
210 D[I,K]=D[I,K]+B[I,J]*A[J,K]
220 NEXT J
225 NEXT K
230 NEXT I
235 PRINT "A X B"
240 FOR I=1 TO M
250 FOR J=1 TO M
260 PRINT C[I,J];
270 NEXT J
280 PRINT
290 NEXT I
300 PRINT
310 PRINT
320 PRINT
325 PRINT "B X A"
330 FOR I=1 TO M
340 FOR J=1 TO N
350 PRINT D[I,J];
360 NEXT J
370 PRINT
380 NEXT I
390 END

```

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RUN

?2,2
A X B
1 2
3 4

B X A
1 2
3 4

30 DATA 6,-2,1,0,1,-1,3,2,1,-2,3,0,4,5,3,-1,0,1

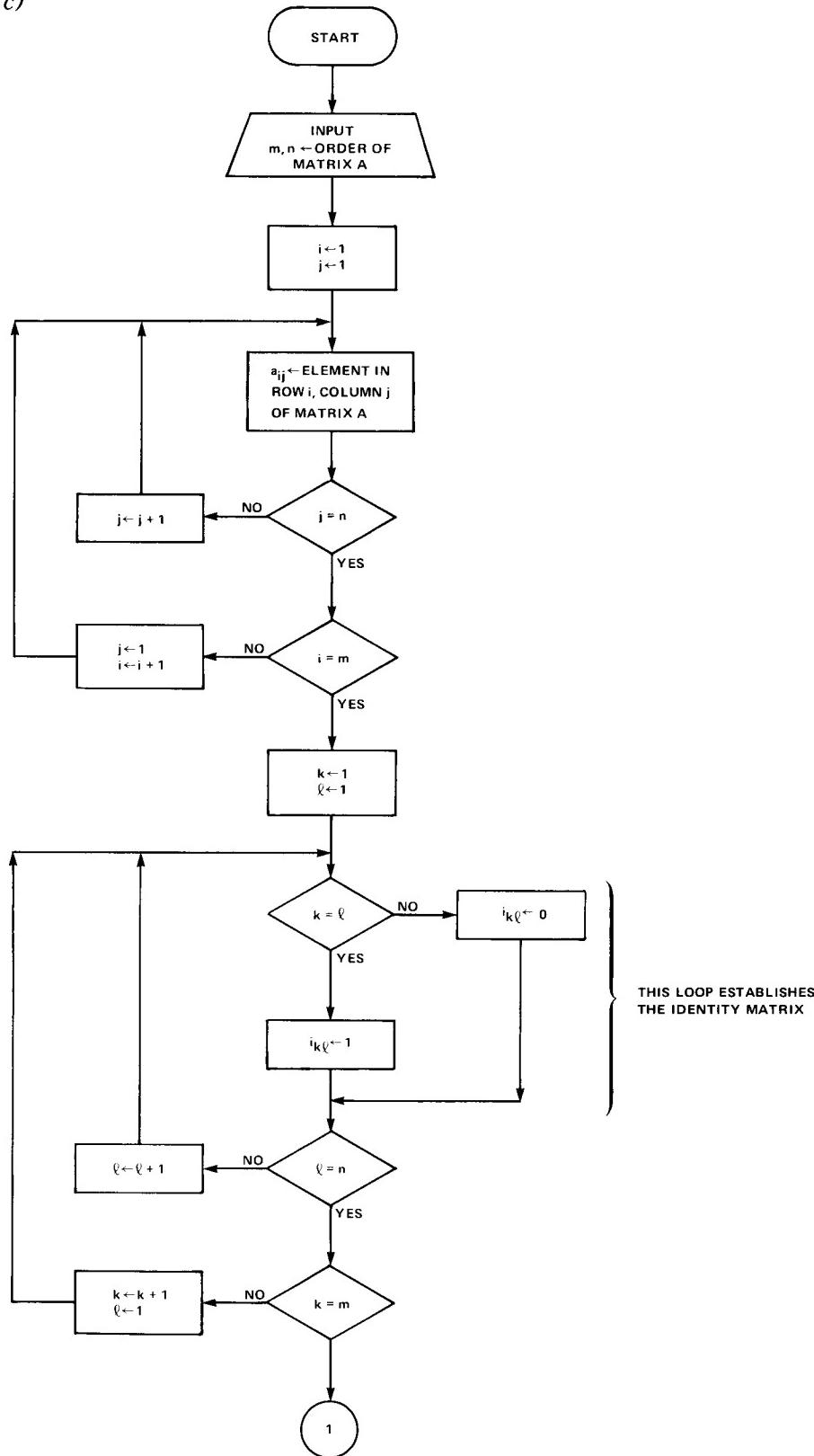
?3,3
A X B
-21 8 -5
 5 5 2
 1 19 7

B X A
-12 7 -5
 33 3 2
 -3 4 0

DONE

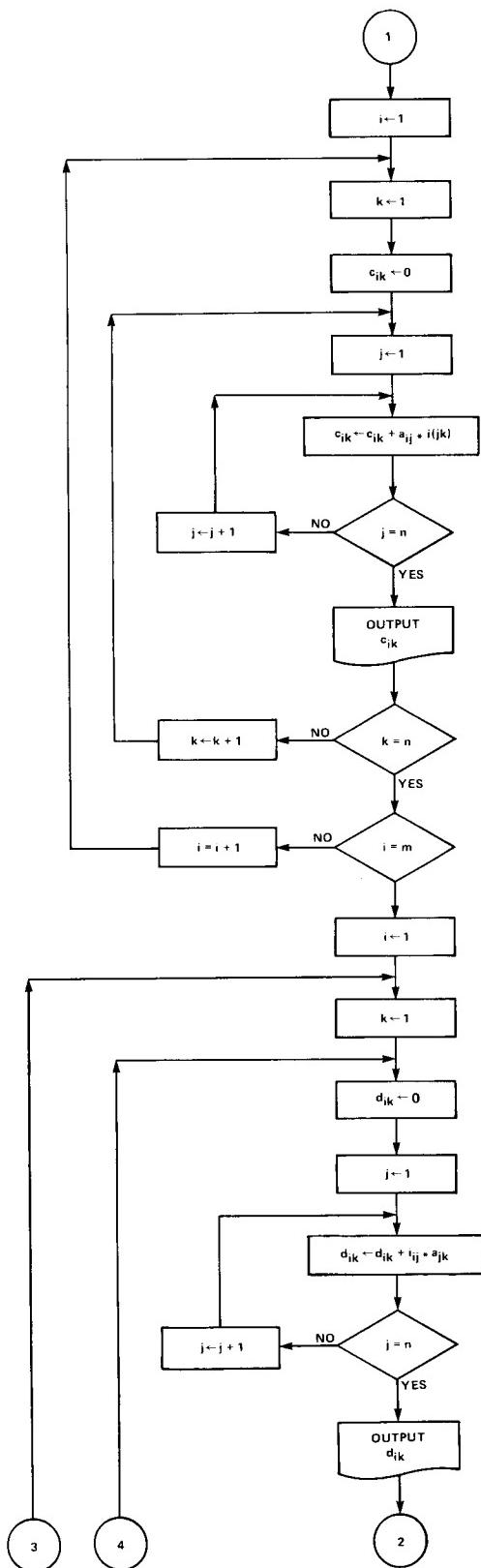
Micro Flow Chart

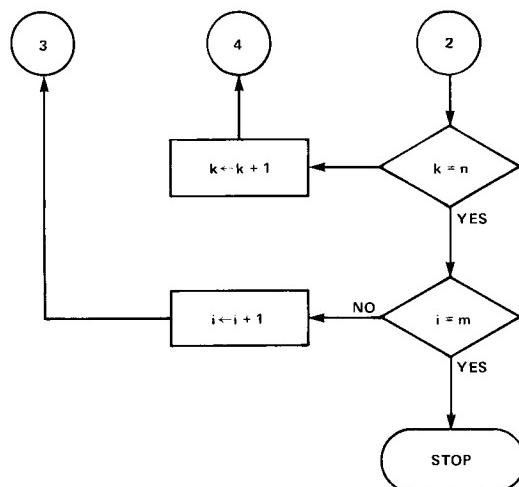
Exercise 6(c)



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Example Program

Exercise 6(c)

```

10 REM--A PROGRAM TO SHOW THAT FOR A GIVEN MATRIX OF REAL NUMBERS
11 REM--THERE EXISTS A MULTIPLICATIVE IDENTITY MATRIX. ON THE
12 REM--DATA LINE ENTER THE ORDER AND ELEMENTS OF THE MATRIX BEING
13 REM--CONSIDERED.
20 DATA 4,4,1,-1,2,-2,3,-3,4,-4,5,-5,6,-6,7,-7,8,-8
30 READ M,N
40 FOR I=1 TO M
50 FOR J=1 TO N
60 READ A[I,J]
70 NEXT J
80 NEXT I
90 FOR K=1 TO M
100 FOR L=1 TO N
110 IF K=L THEN 125
120 LET I[K,L]=0
122 GOTO 130
125 LET I[K,L]=1
130 NEXT L
140 NEXT K
143 PRINT "A X I"
160 FOR I=1 TO M
170 FOR K=1 TO N
180 LET C[I,K]=0
190 FOR J=1 TO N
200 C[I,K]=C[I,K]+A[I,J]*I[J,K]
210 NEXT J
220 PRINT C[I,K];
230 NEXT K
240 PRINT
250 NEXT I
255 PRINT "I X A"
260 FOR I=1 TO M
270 FOR K=1 TO N
280 D[I,K]=0
290 FOR J=1 TO N
300 D[I,K]=D[I,K]+I[I,J]*A[J,K]
310 NEXT J
320 PRINT D[I,K];
330 NEXT K
340 PRINT
350 NEXT I
360 END
  
```

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RUN

A X I
-17 0 22
5 - .83 1
6 -1 0

I X A
-17 0 22
5 - .83 1
6 -1 0

DONE

20 DATA 4,4,1,-1,2,-2,3,-3,4,-4,5,-5,6,-6,7,-7,8,-8
RUN

A X I
1 -1 2 -2
3 -3 4 -4
5 -5 6 -6
7 -7 8 -8

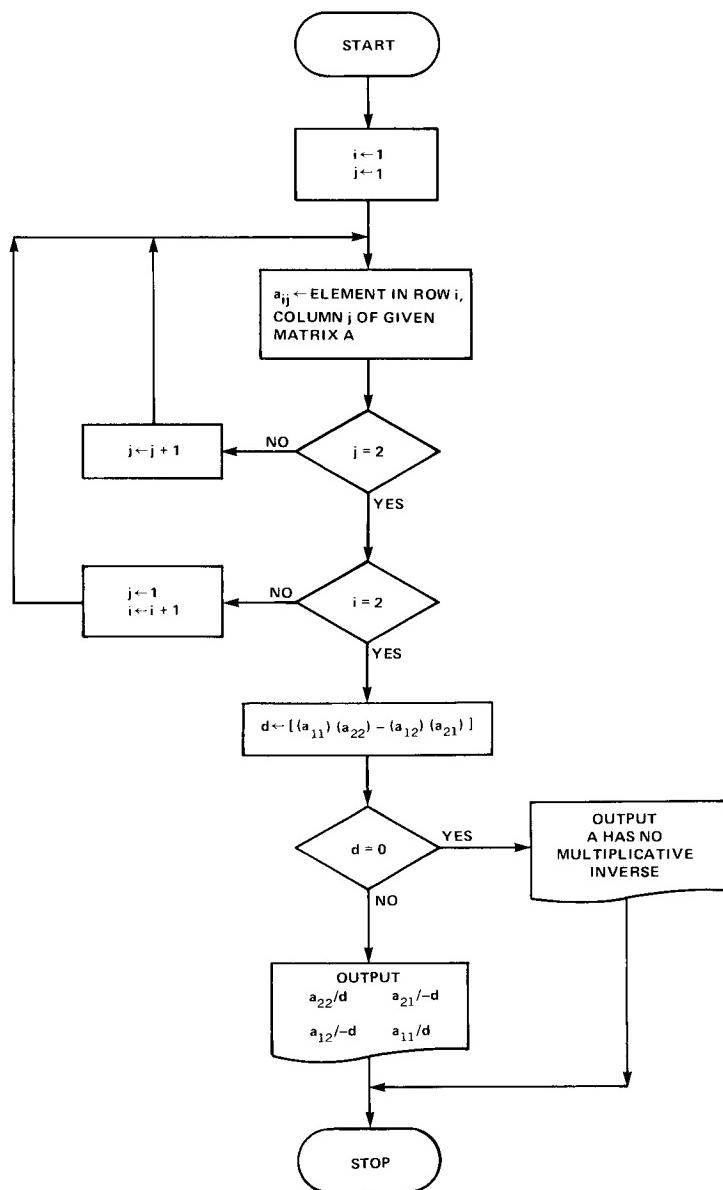
I X A
1 -1 2 -2
3 -3 4 -4
5 -5 6 -6
7 -7 8 -8

DONE

Micro Flow Chart

Exercise 6(d)

In computing the elements of the inverse matrix we get involved in round-off error. Consequently $A \times A^{-1}$ might not equal the identity matrix I . This problem opens the door for a discussion concerning computer error. The following program calls for the inverse matrix to be printed with the elements in decimal form and in rational form.



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Example Program

Exercise 6(d)

```
10  REM-- A PROGRAM TO PRINT THE INVERSE OF A MATRIX
20  FOR I=1 TO 2
30  FOR J=1 TO 2
40  READ A[I,J]
50  NEXT J
60  NEXT I
70  D=A[1,1]*A[2,2]-A[1,2]*A[2,1]
80  IF D=0 THEN 160
90  PRINT A[2,2]/D;A[2,1]/-D
100 PRINT
110 PRINT A[1,2]/-D;A[1,1]/D
120 PRINT """;"/";D,A[2,1];"/";-D"""
130 PRINT A[1,2]"/"-D,A[1,1]"/"D
140 PRINT
150 GOTO 20
160 PRINT "MATRIX 'A' DOES NOT HAVE AN INVERSE."
170 PRINT
180 GOTO 20
190 DATA 17,-3,2,1,6,-5,36,-3
200 END
```

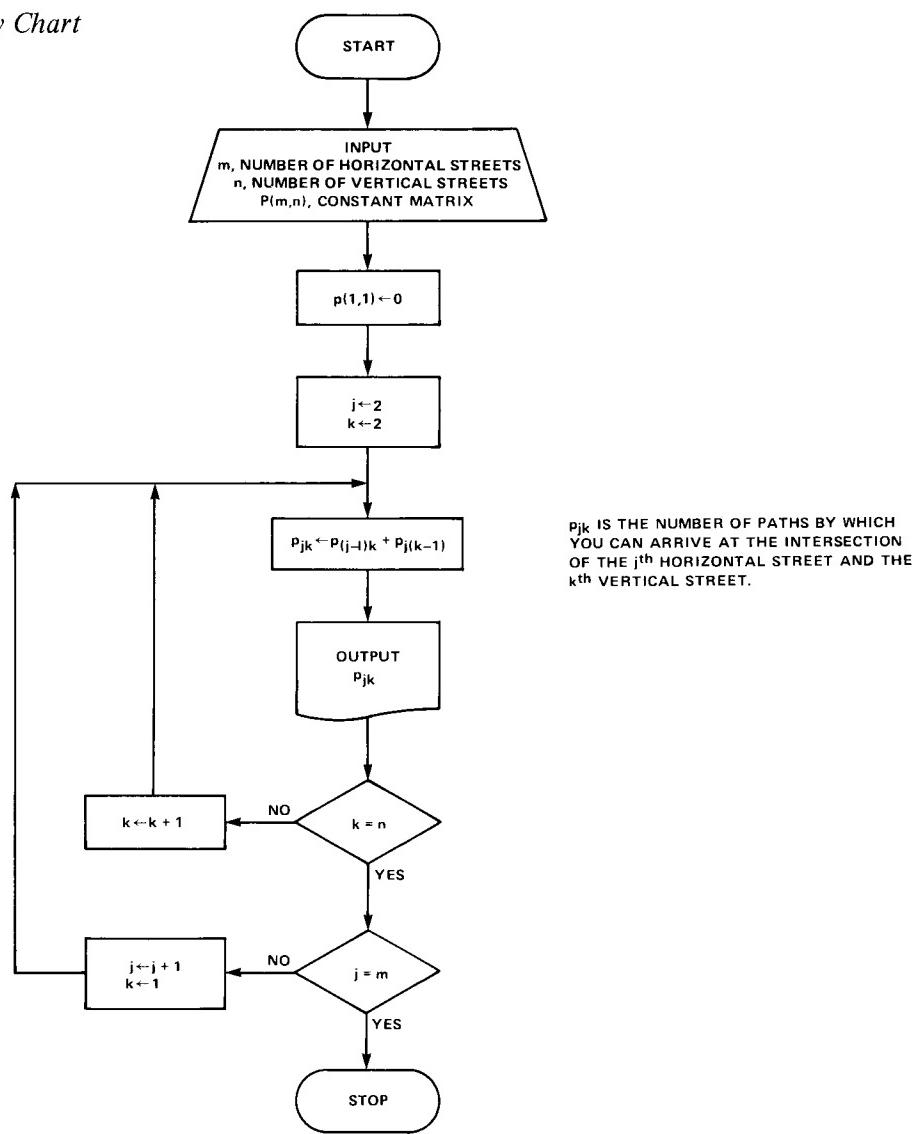
RUN

```
4.34783E-02 -8.69565E-02
.130435 .73913
```

```
1 / 23 2 /-23
-3 /-23 17 / 23
```

MATRIX 'A' DOES NOT HAVE AN INVERSE.

OUT OF DATA IN LINE 40

EXERCISE 7 – A Matrix Application*Micro Flow Chart*

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Example Program

Exercise 7

```
10 REM--A PROGRAM TO DETERMINE THE NUMBER OF PATHS FROM A POINT A
20 REM--TO A POINT B IN A CITY. THE INPUT REQUIRES THE
30 REM--NUMBER OF HORIZONTAL STREETS AND THE NUMBER OF
40 REM--VERTICAL STREETS IN THAT ORDER.
50 INPUT M,N
60 PRINT " THE NUMBER OF PATHS FROM P(1,1) TO:"
70 FOR I=1 TO M
72 FOR J=1 TO N
74 LET P[I,J]=1
76 NEXT J
78 NEXT I
80 LET P[1,1]=0
90 FOR J=2 TO M
100 FOR K=2 TO N
110 LET P[J,K]=P[J-1,K]+P[J,K-1]
120 PRINT TAB(5); "P("; J; ","; K; ") IS"; P[J,K]
130 NEXT K
140 NEXT J
150 END
RUN
```

?4,4
THE NUMBER OF PATHS FROM P(1,1) TO:

P(2 , 2)	IS 2
P(2 , 3)	IS 3
P(2 , 4)	IS 4
P(3 , 2)	IS 3
P(3 , 3)	IS 6
P(3 , 4)	IS 10
P(4 , 2)	IS 4
P(4 , 3)	IS 10
P(4 , 4)	IS 20

DONE

RUN

?6,8

THE NUMBER OF PATHS FROM P(1,1) TO:

P(2 , 2) IS 2
P(2 , 3) IS 3
P(2 , 4) IS 4
P(2 , 5) IS 5
P(2 , 6) IS 6
P(2 , 7) IS 7
P(2 , 8) IS 8
P(3 , 2) IS 3
P(3 , 3) IS 6
P(3 , 4) IS 10
P(3 , 5) IS 15
P(3 , 6) IS 21
P(3 , 7) IS 28
P(3 , 8) IS 36
P(4 , 2) IS 4
P(4 , 3) IS 10
P(4 , 4) IS 20
P(4 , 5) IS 35
P(4 , 6) IS 56
P(4 , 7) IS 84
P(4 , 8) IS 120
P(5 , 2) IS 5
P(5 , 3) IS 15
P(5 , 4) IS 35
P(5 , 5) IS 70
P(5 , 6) IS 126
P(5 , 7) IS 210
P(5 , 8) IS 330
P(6 , 2) IS 6
P(6 , 3) IS 21
P(6 , 4) IS 56
P(6 , 5) IS 126
P(6 , 6) IS 252
P(6 , 7) IS 462
P(6 , 8) IS 792

DONE

MATHEMATICS

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MODULAR ARITHMETIC

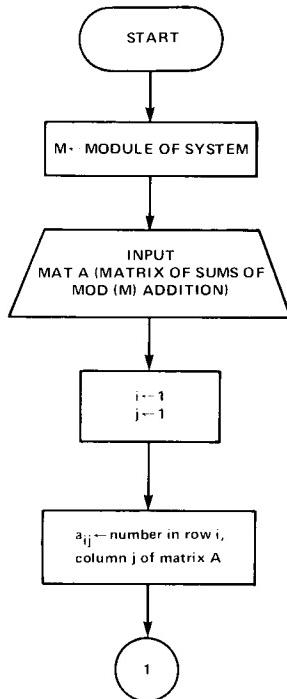
Modular arithmetic should be a satisfying topic of study because it is a finite system, which allows the student to come to concrete, provable conclusions concerning its properties. In the previous section we indicated that matrix theory has many applications. Therefore, it is perfectly acceptable to use the BASIC matrix instructions in the following exercises. Because some users of BASIC may not have these commands available to them the programs are written without them. However, on the listings where to replace statements with MAT statements are indicated.

EXERCISE 8 – Commutativity of Mod (M) Arithmetic

The problem analysis and the flow chart in the student text are rather detailed and will probably need no further explanation

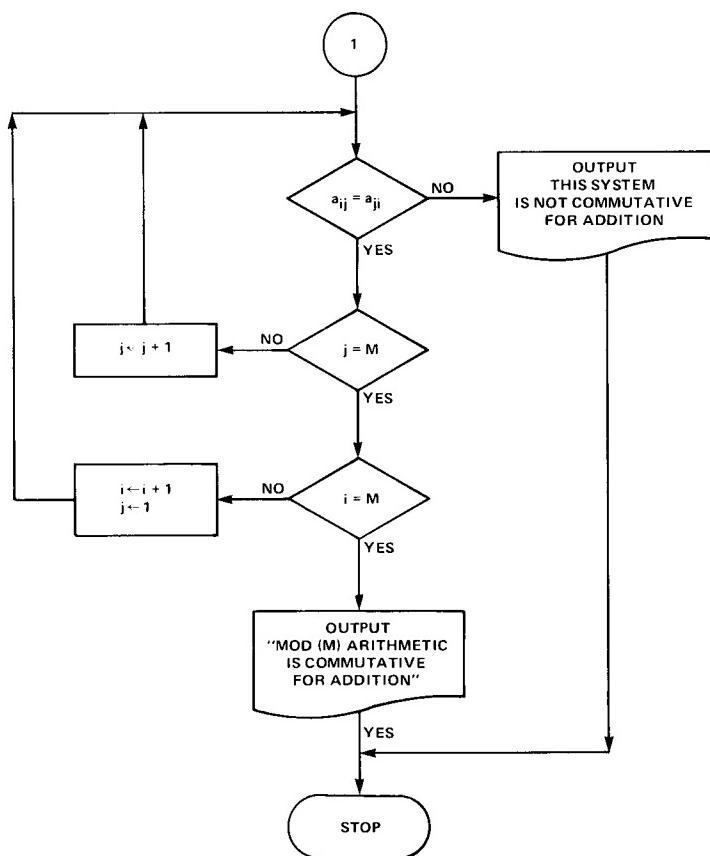
Micro Flow Chart

Exercise 8(a)



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*Example Program**Exercise 8(a) and 8(b)*

```

1 REM--THIS PROGRAM WILL TEST COMMUTATIVITY FOR ADDITION IN MODULAR
2 REM--ARITHMETIC. THE DATA LINE SHOULD CONTAIN THE ELEMENTS FROM
4 REM--ROW 1 DOWN THROUGH THE LAST ROW. THE MATRIX WILL BE
5 REM--OF THE ORDER M, THEREFORE LINE 20 REQUIRES THE MODULE
6 REM--NUMBER BE INPUT FOR M.
20 INPUT M
30 FOR I=1 TO M
32 FOR J=1 TO M
34 READ A[I,J] } 30 MAT READ A[M,M]
36 NEXT J
38 NEXT I
40 DATA 0,1,2,3,1,2,3,0,2,3,0,1,3,0,1,2
50 FOR I=1 TO M
60 FOR J=1 TO M
70 IF A[I,J]=A[J,I] THEN 100
80 PRINT "THIS SYSTEM IS NOT COMMUTATIVE FOR ADDITION."
90 GOTO 130
100 NEXT J
110 NEXT I
120 PRINT "MOD "M" ARITHMETIC IS COMMUTATIVE FOR ADDITION."
130 END
RUN

```

```
?4
MOD 4      ARITHMETIC IS COMMUTATIVE FOR ADDITION.
```

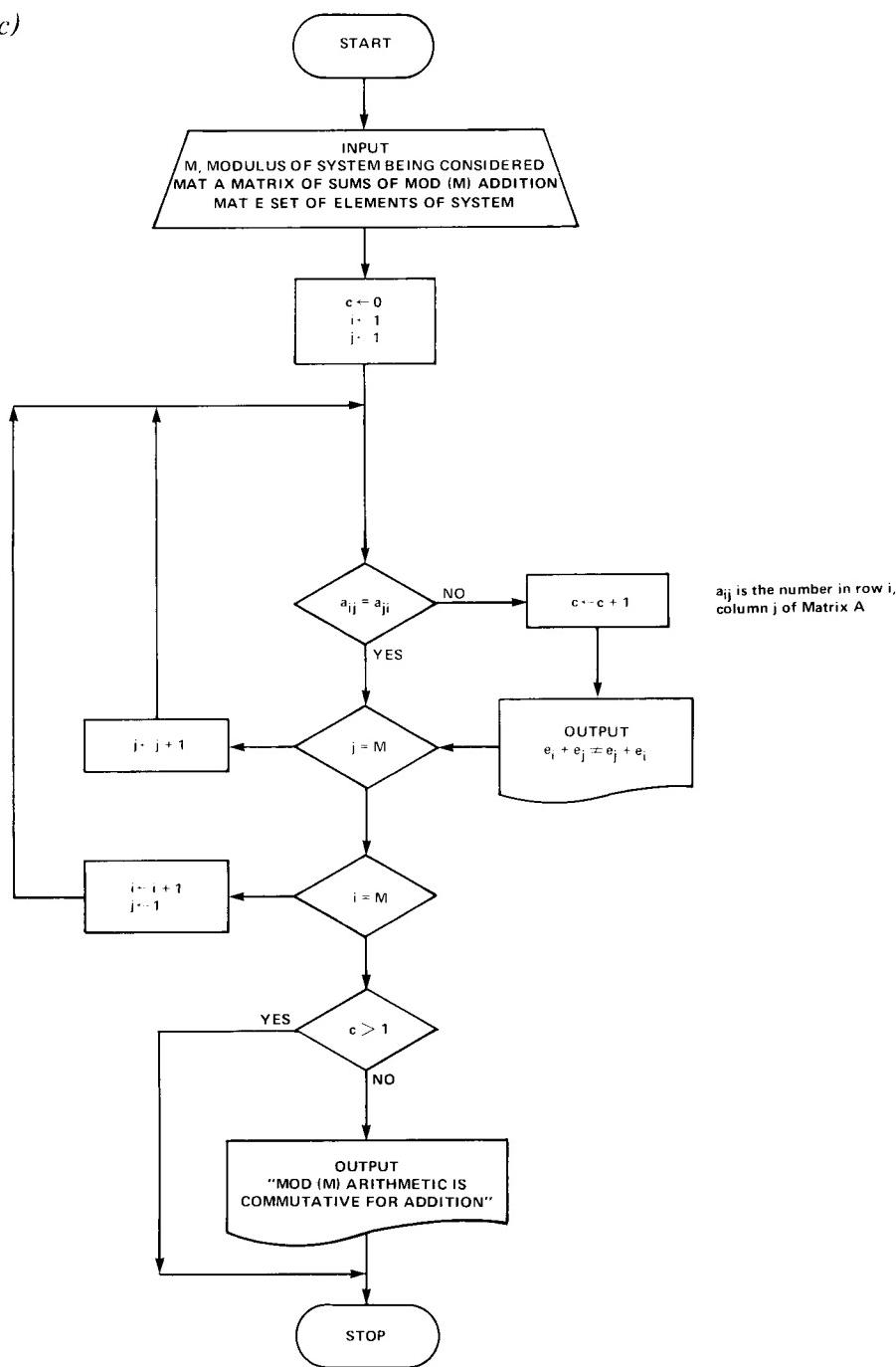
```
DONE
40 DATA 0,1,2,3,1,2,3,0,2,3,0,1,3,0,3,2
RUN
```

```
?4
THIS SYSTEM IS NOT COMMUTATIVE FOR ADDITION.
```

```
DONE
```

Micro Flow Chart

Exercise 8(c)



*Example Program**Exercise 8(c)*

```

10 REM--THIS PROGRAM WILL TEST COMMUTATIVITY FOR ADDITION IN MODULAR
20 REM--ARITHMETIC. ON THE DATA LINE ENTER THE ELEMENTS OF THE
30 REM--MOD IN ORDER FOLLOWED BY THE ELEMENTS OF THE ADDITION MATRIX
40 DATA 0,1,2,3,0,1,2,3,1,2,3,0,2,0,0,1,3,1,2,3
50 REM--BE OF THE ORDER M, THEREFORE LINE 20 REQUIRES THE
60 REM--MODULE NUMBER BE INPUT FOR M.
70 INPUT M
80 FOR I=1 TO M } 80 MAT READ E[1,M]
81 READ E[I,I] } 81 MAT READ E[1,M]
82 NEXT I
83 FOR I=1 TO M }
84 FOR J=1 TO M } 83 MAT READ A[M,M]
85 READ A[I,J] } 85 MAT READ A[M,M]
86 NEXT J
87 NEXT I
90 DATA 0,1,2,3,0,1,2,3,1,2,3,0,2,3,0,1,3,0,1,2
100 C=0
110 FOR I=1 TO M
120 FOR J=1 TO M
130 IF A[I,J]=A[J,I] THEN 180
140 C=C+1
150 IF C>1 THEN 170
160 PRINT "THIS SYSTEM IS NOT COMMUTATIVE FOR THE FOLLOWING CASES:"
170 PRINT E[1,I];"+";E[1,J];"<>";E[1,J];"+";E[1,I]
180 NEXT J
190 NEXT I
200 IF C>1 THEN 230
210 GOTO 220
220 PRINT "MOD "M" ARITHMETIC IS COMMUTATIVE FOR ADDITION."
230 END
RUN

```

```

?4
MOD 4      ARITHMETIC IS COMMUTATIVE FOR ADDITION.

DONE

```

```

40 DATA 0,1,2,3,0,1,2,3,1,2,3,0,2,0,0,1,3,1,2,3
RUN

```

```

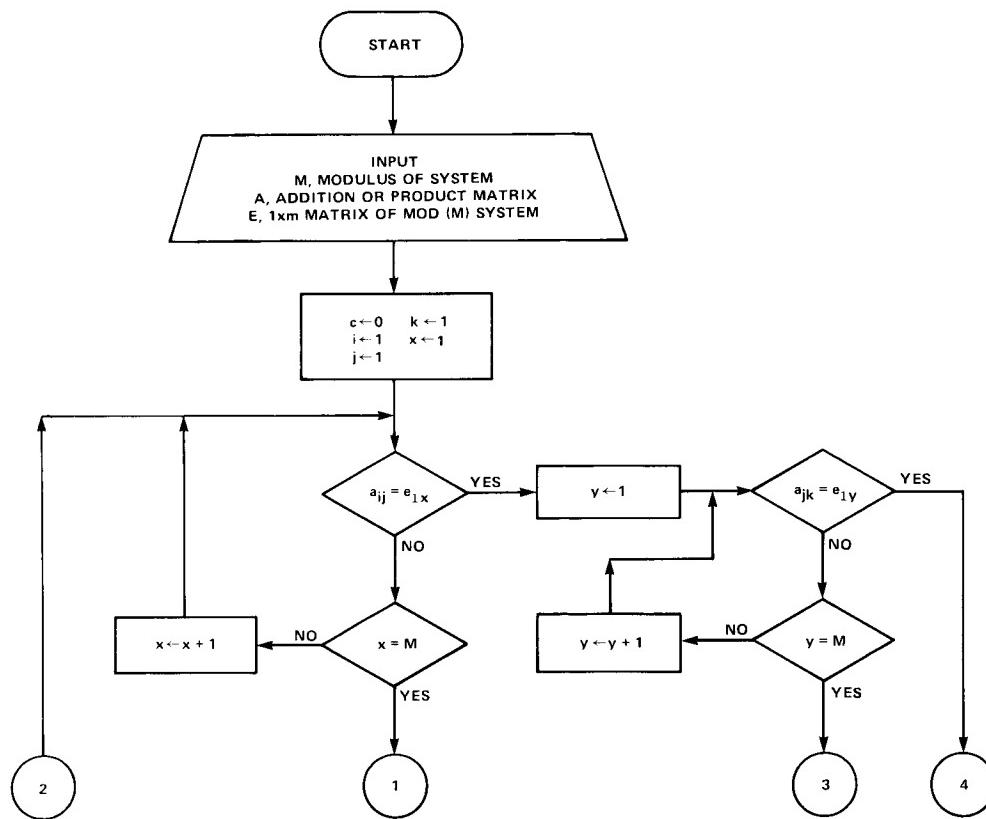
?4
THIS SYSTEM IS NOT COMMUTATIVE FOR THE FOLLOWING CASES:
1   + 2   <> 2   + 1
1   + 3   <> 3   + 1
2   + 1   <> 1   + 2
2   + 3   <> 3   + 2
3   + 1   <> 1   + 3
3   + 2   <> 2   + 3

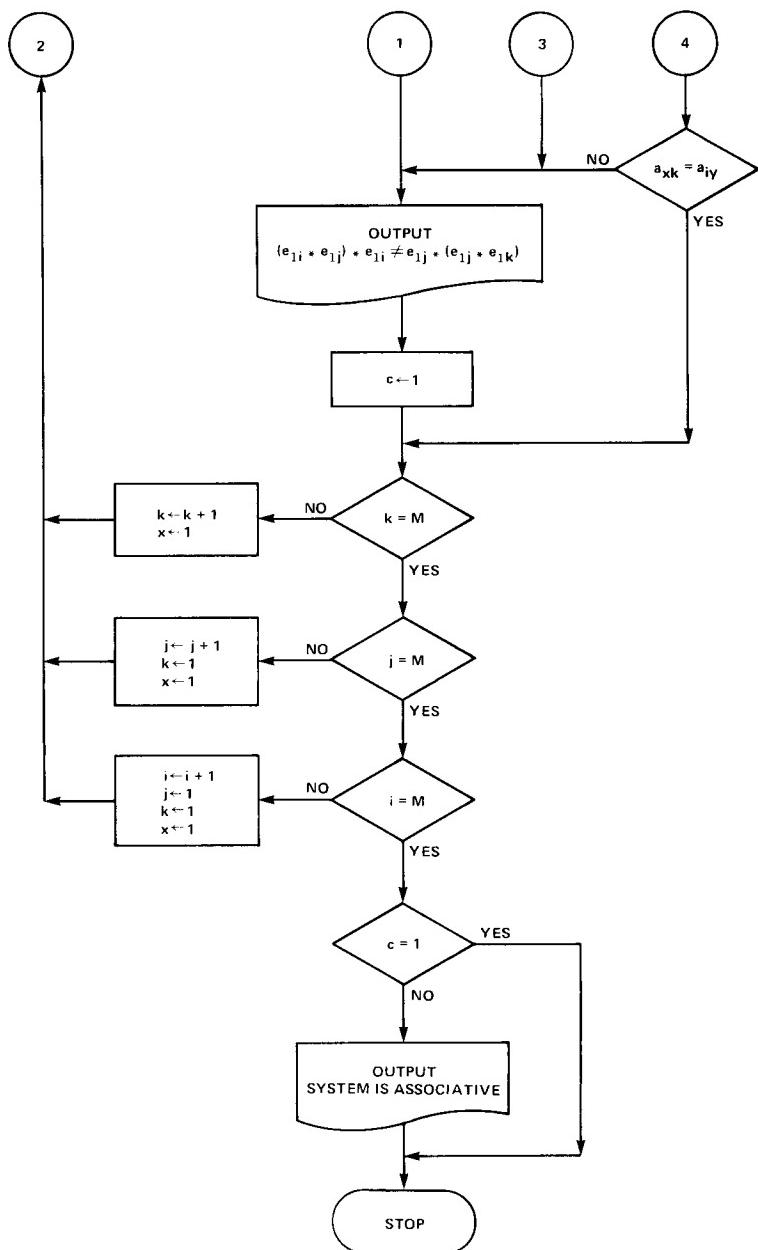
```

DONE

EXERCISE 9 – Associativity for Mod (4) and (5) Arithmetic

Micro Flow Chart





*Example Program**Exercise 9*

```
10 REM--THIS PROGRAM MODELS THE ASSOCIATIVE PROPERTY FOR MODULAR
11 REM--ARITHMETIC (MULTIPLICATION OR ADDITION). ENTER IN THE DATA
12 REM--LINE; THE MODULE OF THE SYSTEM BEING CONSIDERED; THE
13 REM--ELEMENTS OF THE SYSTEM; AND THE ELEMENTS OF THE PRODUCT
14 REM--MATRIX.
20 DATA 4,0,1,2,3,0,0,0,0,1,2,3,0,2,0,2,0,3,2,1
30 READ M
70 FOR I=1 TO M
72 READ E[1,I] } 70 MAT READ E[1,M]
74 NEXT I
80 FOR I=1 TO M
81 FOR J=1 TO M
82 READ A[I,J] } 80 MAT READ A[M,M]
83 NEXT J
84 NEXT I
85 C=0
120 FOR I=1 TO M
130 FOR J=1 TO M
140 FOR K=1 TO M
150 FOR X=1 TO M
160 IF A[I,J]=E[1,X] THEN 260
170 NEXT X
180 PRINT "("E[1,I]"*"E[1,J]"")*"E[1,K]">"E[1,I]"*"E[1,J];
181 PRINT "*E[1,K]"")
190 C=1
200 NEXT K
210 NEXT J
220 NEXT I
230 IF C=1 THEN 320
240 PRINT "MOD(";M;") ARITHMETIC IS ASSOCIATIVE FOR MULTIPLICATION"
250 GOTO 320
260 FOR Y=1 TO M
270 IF A[J,K]=E[1,Y] THEN 300
280 NEXT Y
290 GOTO 180
300 IF A[X,K]=A[I,Y] THEN 200
310 GOTO 180
320 END
```

RUN

MOD(4) ARITHMETIC IS ASSOCIATIVE FOR MULTIPLICATION

DONE

20 DATA 5,0,1,2,3,4,0,0,0,0,0,1,2,3,4,0,2,4,3,3,0,3,1,4,2,0

21 DATA 4,3,2,1

RUN

```
( 2      * 2      )* 3      <> 2      *( 2      * 3      )
( 2      * 2      )* 4      <> 2      *( 2      * 4      )
( 2      * 3      )* 2      <> 2      *( 3      * 2      )
( 2      * 3      )* 3      <> 2      *( 3      * 3      )
( 2      * 3      )* 4      <> 2      *( 3      * 4      )
( 2      * 4      )* 2      <> 2      *( 4      * 2      )
( 3      * 2      )* 3      <> 3      *( 2      * 3      )
( 3      * 4      )* 3      <> 3      *( 4      * 3      )
( 4      * 2      )* 3      <> 4      *( 2      * 3      )
( 4      * 3      )* 3      <> 4      *( 3      * 3      )
```

DONE

20 DATA 5,0,1,2,3,4,0,0,0,0,0,1,2,3,4,0,2,4,1,3,0,3,1,4,2,0

21 DATA 4,3,2,1

RUN

MOD(5) ARITHMETIC IS ASSOCIATIVE FOR MULTIPLICATION

DONE

EXERCISE 10 – Multiplicative Inverse for Mod (M) Arithmetic

Requiring students to model the Multiplicative Inverse Property (MIP) causes them to be aware of two conditions in the definition which are often overlooked or forgotten: (1) The additive identity element (zero) is not included in the MIP generalization, and (2) the generalization requires that each element have a unique inverse.

If the computer system your students are using has matrix manipulation capabilities, this exercise can make use of them.

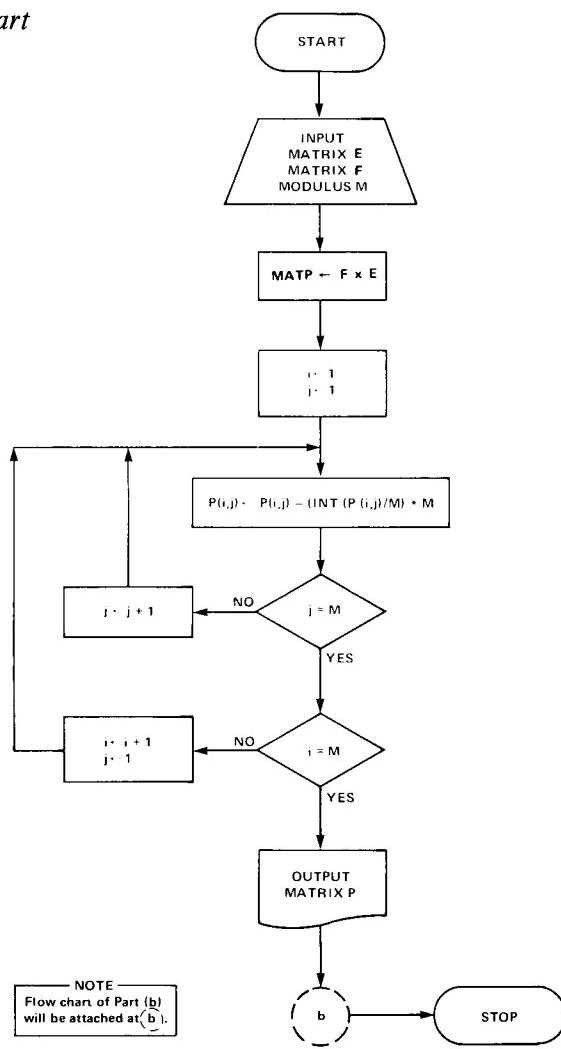
The following flow chart is written for a system with the matrix capabilities. There is a sample program for both systems.

To produce the product matrix P we will let Matrix $F = (0, 1, 2, \dots, M-1)$

$$\text{and Matrix } E = \begin{pmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ M-1 \end{pmatrix} \quad \text{and } P = E \times F$$

Micro Flow Chart

Exercise 10(a)



*Example Program**Exercise 10(a)*

```

10 REM--THIS PROGRAM PRINTS THE PRODUCT MATRIX OF MOD (M)
20 REM--ARITHMETIC. THE DATA LINE REQUIRES THE ELEMENTS FOR
30 REM--MATRIX F WHICH IS A 1 X M MATRIX AND MATRIX E, A M X 1
40 REM--MATRIX OF THE ELEMENTS OF MOD (M) SYSTEM.
50 DATA 4,0,1,2,3,0,1,2,3
60 READ M
70 FOR I=1 TO M
71 FOR J=1 TO M
72 LET P[I,J]=0 } 70 MAT P=ZER[M,M]
73 NEXT J
74 NEXT I
80 FOR I=1 TO M
82 READ E[I,I] } 80 MAT READ E[1,M]
84 NEXT I
90 FOR I=1 TO M
92 READ F[I,1] } 90 MAT READ F[M,1]
94 NEXT I
100 FOR I=1 TO M
102 FOR J=1 TO M
104 LET P[I,J]=P[I,J]+F[I,1]*E[I,J] } 100 MAT P=F*E
106 NEXT J
108 NEXT I
110 FOR I=1 TO M
120 FOR J=1 TO M
130 P[I,J]=P[I,J]-(INT(P[I,J]/M)*M)
140 NEXT J
150 NEXT I
160 FOR I=1 TO M
162 FOR J=1 TO M
164 PRINT P[I,J],
166 NEXT J } 160 MAT PRINT P
167 PRINT
168 PRINT
169 NEXT I
170 END

```

MATHEMATICS

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RUN

0	0	0	0
0	1	2	3
0	2	0	2
0	3	2	1

DONE

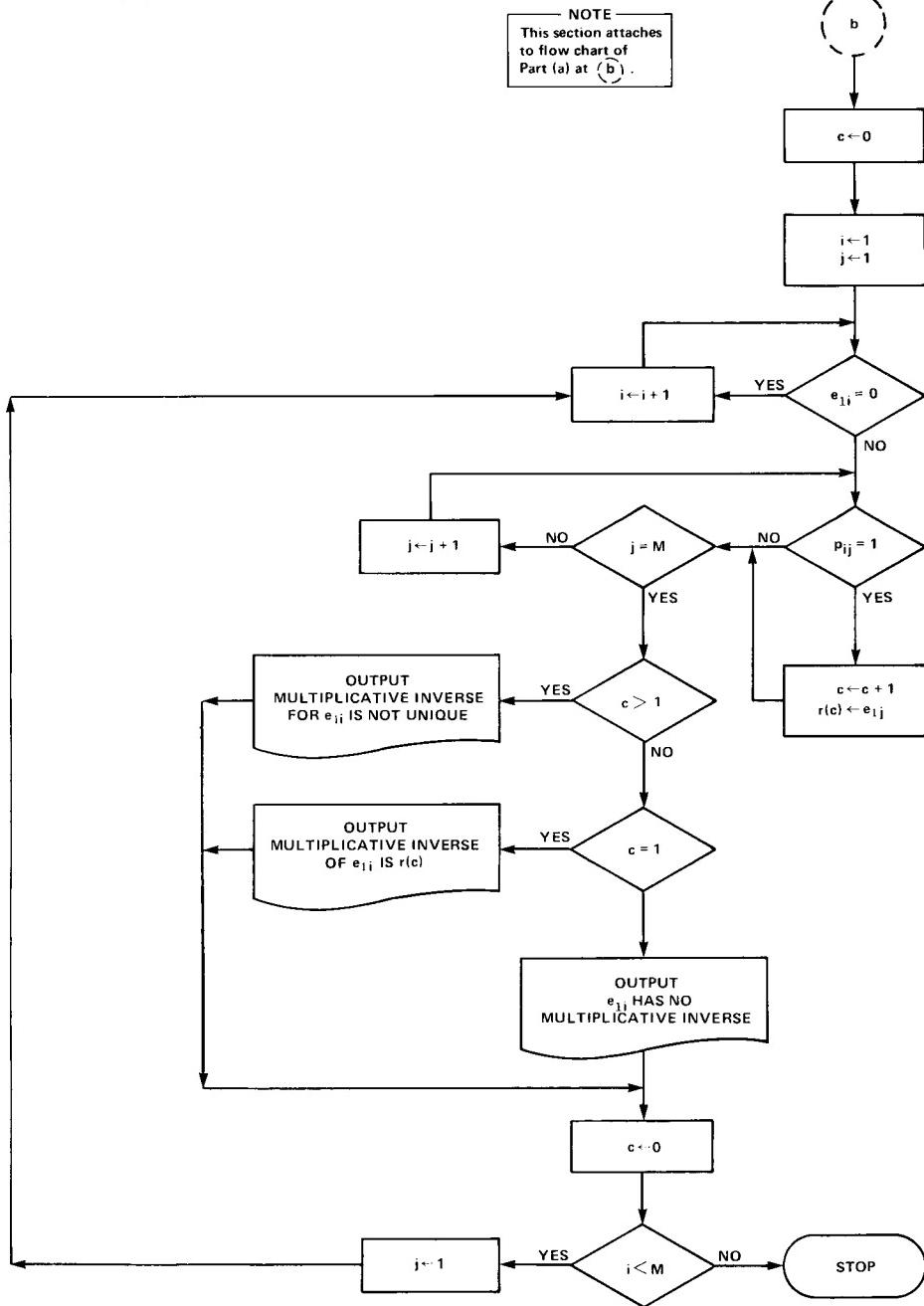
S0 DATA 5,0,1,2,3,4,0,1,2,3,4
RUN

0	0	0	0	0
0	1	2	3	4
0	2	4	1	3
0	3	1	4	2
0	4	3	2	1

DONE

Micro Flow Chart

Exercise 10(b)



*Example Program**Exercise 10(b)*

```

10  REM--THIS PROGRAM WILL DETERMINE IF EACH ELEMENT OF A MOD (M)
20  REM--ARITHMETIC HAS A MULTIPLICATIVE INVERSE
30  DATA 3,0,1,2,0,1,2
40  READ M
50  FOR I=1 TO M
52  FOR J=1 TO M } 50  MAT P=ZER[M,M]
54  LET P[I,J]=0 } 56  NEXT J
56  NEXT I
58  NEXT I
60  FOR I=1 TO M } 62  READ E[I,I] } 60  MAT  READ E[I,M]
62  READ E[I,I] } 64  NEXT I } 60  MAT  READ E[I,M]
64  NEXT I
70  FOR I=1 TO M } 72  READ F[I,I] } 70  MAT  READ F[M,I]
72  READ F[I,I] } 74  NEXT I } 70  MAT  READ F[M,I]
74  NEXT I
80  FOR I=1 TO M
82  FOR J=1 TO M } 84  LET P[I,J]=P[I,J]+F[I,1]*E[I,J] } 80  MAT P=F*E
86  NEXT J
88  NEXT I
90  FOR I=1 TO M
100 FOR J=1 TO M
110 P[I,J]=P[I,J]-(INT(P[I,J]/M)*M)
120 NEXT J
130 NEXT I
140 LET C=0
150 FOR I=1 TO M
160 FOR J=1 TO M
170 IF E[I,I]=0 THEN 330
180 IF P[I,J]=1 THEN 210
190 GOTO 230
200 IF J=M THEN 240
210 LET C=C+1
220 LET R[C]=E[I,J]
230 NEXT J
240 IF C>1 THEN 280
250 IF C=1 THEN 300
260 PRINT E[I,I];"HAS NO MULTIPLICATIVE INVERSE."
270 GOTO 310
280 PRINT "MULTPLICATIVE INVERSE OF" E[I,I] "IS NOT UNIQUE."
290 GOTO 310
300 PRINT "MULTPLICATIVE INVERSE OF " E[I,I] " IS " R[C]
310 C=0
320 LET J=1
330 NEXT I
340 END

```

RUN

MULTIPLICATIVE INVERSE OF 1 IS 1
MULTIPLICATIVE INVERSE OF 2 IS 2

DONE

30 DATA 4,0,1,2,3,0,1,2,3
RUN

MULTIPLICATIVE INVERSE OF 1 IS 1
2 HAS NO MULTIPLICATIVE INVERSE.
MULTIPLICATIVE INVERSE OF 3 IS 3

DONE

30 DATA 5,0,1,2,3,4,0,1,2,3,4
RUN

MULTIPLICATIVE INVERSE OF 1 IS 1
MULTIPLICATIVE INVERSE OF 2 IS 3
MULTIPLICATIVE INVERSE OF 3 IS 2
MULTIPLICATIVE INVERSE OF 4 IS 4

DONE

30 DATA 6,0,1,2,3,4,5,0,1,2,3,4,5
RUN

MULTIPLICATIVE INVERSE OF 1 IS 1
2 HAS NO MULTIPLICATIVE INVERSE.
3 HAS NO MULTIPLICATIVE INVERSE.
4 HAS NO MULTIPLICATIVE INVERSE.
MULTIPLICATIVE INVERSE OF 5 IS 5

DONE

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NOTES

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